Greenberger-Horne-Zeilinger generation protocol for N superconducting transmon qubits capacitively coupled to a quantum bus

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We propose a circuit quantum electrodynamics (QED) realization of a protocol to generate a Greenberger-Horne-Zeilinger (GHZ) state for N superconducting transmon qubits homogeneously coupled to a superconducting transmission line resonator in the dispersive limit. We derive an effective Hamiltonian with pairwise qubit exchange interactions of the XY type, $\tilde{g}(XX+YY)$, that can be globally controlled. Starting from a separable initial state, these interactions allow to generate a multi-qubit GHZ state within a time $t_{\mathrm{GHZ}} \sim \tilde{g}^{-1}$. We discuss how to probe the non-local nature and the genuine N-partite entanglement of the generated state. Finally, we investigate the stability of the proposed scheme to inhomogeneities in the physical parameters.

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I. INTRODUCTION

Entangled quantum states are one of the essential resources for quantum information processing. They are necessary for the realization of quantum communication and the most important computational tasks. Many efforts have been devoted to the elaboration of physical systems enabling the generation and the control of such states. In particular, different types of superconducting qubits are promising candidates to solve this problem. Until recently limited to two qubits[1–4], efforts to entangle superconducting qubits have lately reached a new milestone with the experimental demonstration of three-qubit entanglement[5,6].

In the present paper we consider transmon qubits[7,8] in a circuit quantum electrodynamics architecture[9–12] and present a way to generate GHZ states[13], i.e., maximally entangled states. Although the mathematical description of multipartite entanglement for more than three qubits is still debated[14–16], GHZ states remain paradigmatic entangled states which are, in particular, useful for fault-tolerant quantum computing or quantum secret sharing[17]. So far, many different protocols have been proposed to generate such states in circuit QED setups[18–23]. Some of them are of probabilistic nature, i.e., if a measurement on the N-qubit system has a specific result, the system is known to be in a GHZ state after the measurement[19–21]. In Ref. 23, a Mølmer-Sørensen type[24] one-step scheme to generate GHZ states both for superconducting flux qubits and charge qubits was proposed. The procedure is independent of the initial state of the quantum bus and works in the presence of multiple low-excitation modes. However, higher excitation modes of the quantum bus will introduce inhomogeneity because of the shorter wavelengths of the higher modes and decrease the GHZ fidelity. Moreover, uncontrolled dissipation might be coupled through the higher excitation modes and induce extra noise. It would be ideal to devise a GHZ generation scheme that, while keeping the one-step, deterministic nature, would involve only a single mode of the quantum bus mediating the qubit interaction.

For this purpose, in the present paper, we consider N superconducting transmon qubits homogeneously coupled to a superconducting transmission line resonator in the dispersive limit, i.e., the architecture realized in a number of experiments[3,5,11,25–27]. We show that the system is characterized by effective qubit exchange interactions of XY type that can be globally controlled. Starting from a separable initial state, these interactions allow to generate a GHZ state in a deterministic one-step procedure. We discuss how to probe the non-local nature and the genuine N-partite entanglement of the generated state and investigate the stability of the proposed scheme to inhomogeneities in the physical parameters. In contrast to Ref. 23, the qubit-resonator interaction does not commute with the free Hamiltonian, and the qubit frequencies are tuned close to one resonator mode. The time evolution of the system is described by an effective Hamiltonian which allows a direct implementation of the Mølmer-Sørensen idea. Our scheme is the first one-step deterministic generation protocol of GHZ states which could be possibly implemented in the currently available circuit QED design.

The paper is organized as follows: in Section II we derive an effective Hamiltonian for N transmon qubits capacitively coupled to a superconducting transmission line resonator in the dispersive regime. In Section III we describe the protocol for generating GHZ states in our system. In Section IV, we discuss ways to confirm the N-partite nature of the entanglement in the generated states, and in Section V we study the effects of non-ideal physical parameters like inhomogeneities in the qubit-resonator coupling constants.

II. FULLY CONNECTED NETWORK OF TRANSMON QUBITS IN THE DISPERSIVE LIMIT

We propose a solid-state implementation, based on an architecture of superconducting transmon qubits capacitively coupled to a quantum bus and derive an effective Hamiltonian for the system, which exhibits the appropriate XY exchange interaction.

Transmon qubits consist of a superconducting island connected to a superconducting electrode through a Josephson tunnel junction with capacitance C_J and an extra shunting capacitance C_B. A gate voltage V_g is applied to the island via a gate capacitance C_g, allowing to tune the dimensionless gate charge $n_g = C_g V_g/(2e)$. The system is characterized by the charging energy $E_C = e^2/(2C_\Sigma)$, where $C_\Sigma = C_g + C_J + C_B$ is the total capacitance of the island, and the Josephson energy E_J of the tunnel junction.

Such Josephson junction based qubits behave effectively as quantum two-level systems in different regimes, categorized by the ratio E_J/E_C. We will focus on the so-called transmon regime, when $E_J/E_C \sim 50 - 100$. In this case the Hamiltonian of a single transmon qubit $\mathcal{H}_{\rm qb}$ can be expressed as

$$\mathcal{H}_{\rm qb} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi}\,. \tag{1}$$

In the following we assume that the Josephson junctions form a dc-SQUID i.e., E_J is tunable by an external applied magnetic flux $\Phi_{\rm ext}$ allowing to control independently each qubit. In this case $C_\Sigma = C_g + 2C_J^{(1)} + C_B$ and $E_J = 2\tilde{E}_J \cos(\pi\Phi_{\rm ext}/\Phi_0)$ with $C_J^{(1)}$ and $\tilde{E}_J$ the capacitance and the Josephson energy of a single junction.

If a qubit is capacitively coupled to a superconducting transmission line resonator, C_g is now the capacitance between the superconducting island and the resonator. In that particular situation the gate voltage involves a dc-part and an extra term depending on the state of the resonator, $V_g = V_g^{\rm dc} + V(x)$. Therefore the interaction with the resonator appears via the gate charge n_g, which implicitly includes the voltage $V(x)$. Transmon qubits are more robust to $1/f$-noise than charge qubits due to their exponentially suppressed charge dispersion[8]. However, we assume that the gate of each qubit can be controlled separately by microwave pulses in order to perform single qubit quantum gates.

For simplicity we consider the qubits to be coupled to a single mode of the resonator. This is a reasonable assumption if the qubits are nearly resonant with only one mode. Since higher modes have frequencies which are multiples of the fundamental frequency, we can tune the qubit transition frequencies such that the detuning with respect to one mode of the resonator is one order of magnitude smaller than the detuning to all the other modes. Under these conditions we can realize the dispersive limit for a single mode of the resonator and neglect the influence of higher modes, as is the case in experiments using one transmon qubit[28].

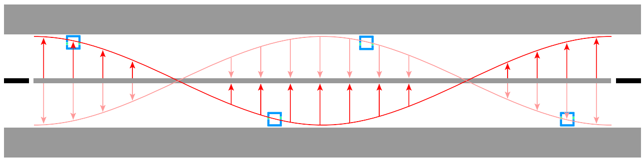

FIG. 1: (Color online) Sketch of a possible coplanar geometry for the proposed device with $N = 4$ qubits. Qubits (small blue squares) are placed around the maxima of the electrical field amplitude (red line), i.e. near the center and the ends of the (quasi-)one-dimensional resonator (gray strip). The second mode of the electrical field (red arrows) mediates the qubit-qubit interaction. Input and output ports of the resonator are drawn in black.

For instance the qubits could be mainly coupled to the second mode if they are placed near the ends or the center of the resonator, that is the positions where the electrical field amplitude is maximal. Such a possible geometry is sketched in Fig. 1. Following the procedure of canonical quantization of a (quasi-) one-dimensional superconducting resonator[9], the voltage across the resonator is given by

$$V(x) = \sqrt{\frac{\omega_r}{L_0 c}} \cos\left(\frac{2\pi x}{L_0}\right)(a + a^\dagger)\,. \tag{2}$$

The length of the resonator is L_0 and its resonance frequency $\omega_r = 2\pi/\sqrt{L_0^2 lc}$ depends on its capacity c and inductance l per unit length. The position along the resonator is denoted by $x \in [-L_0/2, L_0/2]$ and $a(a^\dagger)$ represent bosonic annihilation (creation) field operators.

Following Ref. 7, the system, composed of the resonator and N transmon qubits, can be described with a generalized Jaynes-Cummings Hamiltonian. This Hamiltonian is expressed in the basis of transmon eigenstates $|j\rangle_q$, where the indices q label the transmon qubits, and for readability we define the operators $\Pi_j^{(q)} = |j\rangle_q\langle j|_q$, $\sigma_{j,-}^{(q)} = |j\rangle_q\langle j+1|_q$, $\sigma_{j,+}^{(q)} = |j+1\rangle_q\langle j|_q$ and set $\hbar = 1$,

$$\mathcal{H} = \omega_r a^\dagger a + \sum_{q=1}^{N}\sum_j \left[\omega_j^{(q)}\Pi_j^{(q)} + \left(g_j^{(q)}\, a\, \sigma_{j,+}^{(q)} + \text{H.c}\right)\right]. \tag{3}$$

The qubits frequencies $\omega_j^{(q)}$ are presumed to be tunable through external magnetic fields $\Phi_{\rm ext}^{(q)}$, changing the effective Josephson energies of the qubits $E_J^{(q)} = 2\tilde{E}_J^{(q)}\cos(\pi\Phi_{\rm ext}^{(q)}/\Phi_0)$, and the coupling frequencies $g_j^{(q)}$ depend on the position of the qubits. Invoking the rotating-wave approximation, we have neglected rapidly oscillating terms. In the transmon regime, we can only keep transmon-resonator coupling coefficients for neighboring levels, since terms like $|i\rangle_q\langle j|_q$ for $|i-j| > 1$ are comparatively small. Moreover in the large E_J/E_C limit asymptotic expression can be obtained for $\omega_j^{(q)}$ and $g_j^{(q)}$

in first order perturbation theory[7],

$$\omega_j^{(q)} \simeq \sqrt{8E_C^{(q)}E_J^{(q)}}\left(j+\frac{1}{2}\right) - \frac{E_C^{(q)}}{12}(6j^2+6j+3),$$
$$g_j^{(q)} \simeq g_0^{(q)}\sqrt{j+1}\cos\left(\frac{2\pi x_q}{L_0}\right), \tag{4}$$
$$g_0^{(q)} \simeq -i\frac{eC_g^{(q)}}{C_\Sigma^{(q)}}\left(\frac{E_J^{(q)}}{2E_C^{(q)}}\right)^{1/4}\sqrt{\frac{\omega_r}{L_0 c}}.$$

This form of the coupling frequencies $g_j^{(q)}$ describes the situation shown in Fig. 1. The amplitudes of these coupling coefficients $g_j^{(q)}$ can be assumed to be approximately homogeneous if the positions x_q of the qubits satisfy $|x_q/L_0| \simeq 0$ or $1/2$, since the electrical field amplitude decreases quadratically with the distance from its maxima and since the size of the qubits is typically much smaller than the resonator wavelength in realistic systems. However even if close to the center or the ends of the resonator, the qubits should be placed sufficiently far apart to reduce direct inductive or capacitive qubit-qubit coupling. There are also other positions that the qubits can be placed in (e.g. nodes of higher modes). However, the homogeneity of the coupling constants is important in our approach and should be taken care of.

In the so-called dispersive regime $|g_j^{(q)}/\Delta_j^{(q)}| \ll 1$, when transitions frequencies of the transmon qubits $\omega_{j,j+1}^{(q)}$ are detuned from the resonator frequency ω_r, excitations of the resonator are virtual and the latter will rather act as a quantum bus mediating effective qubit-qubit interactions. The transition frequencies of the transmon qubits are defined as $\omega_{j,j+1}^{(q)} = \omega_{j+1}^{(q)} - \omega_j^{(q)}$ and their respective detuning as $\Delta_j^{(q)} = \omega_{j,j+1}^{(q)} - \omega_r$. In this regime, eliminating the direct interaction between resonator and transmon qubits to lowest order in $g_j^{(q)}/\Delta_j^{(q)}$, we exhibit an effective qubit-qubit interaction. This can be seen by performing the canonical transformation $e^S\mathcal{H}e^{-S}$, where

$$S = \sum_{q=1}^{N}\sum_j\left(\frac{g_j^{(q)}}{\Delta_j^{(q)}}\,a\,\sigma_{j,+}^{(q)} - \text{H.c}\right). \tag{5}$$

Keeping terms up to second order in g_j/Δ_j, we obtain.

$$\begin{aligned} &e^S\mathcal{H}e^{-S} \\ &\simeq \left(\omega_r + \sum_{q=1}^{N}\left[-\chi_0^{(q)}\Pi_0^{(q)} + \sum_{j\geq 1}\left(\chi_{j-1}^{(q)} - \chi_j^{(q)}\right)\Pi_j^{(q)}\right]\right)a^\dagger a \\ &\quad + \sum_{q=1}^{N}\left[\omega_0^{(q)}\Pi_0^{(q)} + \sum_{j\geq 1}\left(\omega_j^{(q)} + \chi_{j-1}^{(q)}\right)\Pi_j^{(q)}\right] \\ &\quad + \sum_{q=1}^{N}\left[aa\sum_j \eta_j^{(q)}\sigma_{j+1,+}^{(q)}\sigma_{j,+}^{(q)} + \text{H.c}\right] \\ &\quad + \sum_{q\neq q'}\sum_{j,j'}\left[\frac{\tilde{g}_{jj'}^{(qq')}}{2}\left(\sigma_{j,+}^{(q)}\sigma_{j',-}^{(q')} + \sigma_{j,-}^{(q)}\sigma_{j',+}^{(q')}\right)\right], \end{aligned} \tag{6}$$

Here the ac-Stark shifts $\chi_j^{(q)}$, the two-photon transition rates $\eta_j^{(q)}$ and the effective qubit-qubit coupling coefficient $\tilde{g}_{jj'}^{(qq')}$ are given by

$$\begin{aligned} \chi_j^{(q)} &= \frac{|g_j^{(q)}|^2}{\Delta_j^{(q)}}, \\ \eta_j^{(q)} &= \frac{1}{2}\frac{g_j^{(q)}g_{j+1}^{(q)}}{\Delta_j^{(q)}\Delta_{j+1}^{(q)}}\left(\omega_{j,j+1}^{(q)} - \omega_{j+1,j+2}^{(q)}\right), \\ \tilde{g}_{jj'}^{(qq')} &= \left|g_j^{(q)}g_{j'}^{*(q')}\right|\frac{\Delta_j^{(q)} + \Delta_{j'}^{(q')}}{2\Delta_j^{(q)}\Delta_{j'}^{(q')}}. \end{aligned} \tag{7}$$

Two-photon transitions can be safely neglected since the parameters $\eta_j^{(q)}$ are small in the dispersive regime. An effective Hamiltonian $\mathcal{H}_{\text{eff}}$ is now obtained by restricting our Hilbert space to the computational subspace, that is the first two levels of each transmon qubit $\{|0\rangle, |1\rangle\}^{\otimes N}$. In principle, the qubit-qubit interaction couples any states of the qubits with more than one excitations to states that do not belong to the computational subspace (e.g. for $N = 3$, the state $|110\rangle$ or $|111\rangle$ will be coupled to $|020\rangle$ or $|021\rangle$). However, the amplitudes for these mixing processes of computational states with such non-computational states are of order $g^2/(E_C\Delta)$ and will be neglected[29]. Under these conditions,

$$\begin{aligned} \mathcal{H}_{\text{eff}} = &\left(\omega + \sum_q \chi^{(q)}\sigma_z^{(q)}\right)a^\dagger a + \sum_q \frac{\tilde{\omega}_{01}^{(q)}}{2}\sigma_z^{(q)} \\ &+ \sum_{q,q'}\frac{\tilde{g}_{00}^{(qq')}}{4}\left(\sigma_x^{(q)}\sigma_x^{(q')} + \sigma_y^{(q)}\sigma_y^{(q')}\right), \end{aligned} \tag{8}$$

where $\chi^{(q)} = \chi_0^{(q)} - \chi_1^{(q)}/2$, $\sigma_z = \Pi_1 - \Pi_0$, $\sigma_x = \sigma_+ + \sigma_-$, $\sigma_y = -i(\sigma_+ - \sigma_-)$. The resonator and qubit frequencies get slightly renormalized, that is $\omega = \omega_r - \sum_q \chi_1^{(q)}/2$

and $\tilde{\omega}_{01}^{(q)} = \omega_{01}^{(q)} - \chi_0^{(q)}$. The Hamiltonian has the desired YX-form, provided that all qubits have identical parameters: that is all qubit and coupling frequencies are homogeneous, $\tilde{\omega}_{01}^{(q)} = \Omega$, $|g_0^{(q)}| = g$, $\Delta_0^{(q)} = \Delta$ and $\tilde{g}_{00}^{(qq')} = \chi_0^{(q)} = \tilde{g} = g^2/\Delta$. Using Eq. (4) we infer that $\chi^{(q)} = \chi = -\tilde{g}E_C/(\Delta - E_C) < \tilde{g}$, where $E_C = \omega_{01} - \omega_{12}$ is the weak anharmonicity of the transmon qubits. As mentioned earlier in Eq. (4) the qubit transition frequencies can be made homogeneous by tuning the flux biases $\Phi_{\text{ext}}^{(q)}$. From now on we assume the $g_j^{(q)}$ are homogeneous. This is motivated by a promising new transmon architecture with tunable coupling that has been proposed recently[31]. Inhomogeneous coupling constants will be discussed in Sec. V.

Previous GHZ state generation protocols based on homodyne measurement of the transmission line[19–21] neglected the effective exchange interaction because of the large differences in qubit frequencies. In our case, the qubit frequencies $\omega_{01}^{(q)}$ are tuned to be identical using the flux biases, and this effective interaction plays a significant role in the generation of the GHZ state in a one-step procedure as shown below.

If the qubit and coupling frequencies are homogeneous, the total spin operators $\hat{J}_{x,y,z} = \frac{1}{2}\sum_q \sigma_{x,y,z}^{(q)}$ and their corresponding Casimir operator $\hat{J}^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2$ can be used to write the effective Hamiltonian in a very convenient form,

$$\mathcal{H}_{\text{eff}} = \omega a^\dagger a + \tilde{g}\,\hat{J}^2 + (\Omega + 2\chi a^\dagger a)\hat{J}_z - \tilde{g}\,\hat{J}_z^2. \tag{9}$$

Evidently, $\mathcal{H}_{\text{eff}}$ is diagonal in the basis $|J, J_z\rangle \otimes |n\rangle$, where the states $|J, J_z\rangle$, describing the states of the N qubits, are the eigenstates of the operators $\hat{J}^2$ and $\hat{J}_z$ with respective eigenvalues $J(J+1)$ and J_z. The states $|n\rangle$, describing the state of the resonator, are eigenstates of $a^\dagger a$ with eigenvalue $n \geq 0$. Since $[\mathcal{H}, \hat{J}^2] = 0$, any eigenstates of $\hat{J}^2$ will remain so under the action of this Hamiltonian. In the following, we will restrict ourselves to such states with $J = N/2$. For example states with all spins aligned in a particular direction belong to this type and are therefore an appropriate choice for the initial state. Setting $J = N/2$ in what follows, we denote $|J = N/2, J_z\rangle$ by $|J_z\rangle$. The eigenstates of $\mathcal{H}_{\text{eff}}$ are $|J_z\rangle \otimes |n\rangle$ with eigenvalues $\varepsilon(n, J_z) = \omega n + \tilde{g}(N/2+1)N/2 + (\Omega + 2\chi n)J_z - \tilde{g}J_z^2$.

III. PROTOCOL FOR GENERATING GHZ STATES

The effective Hamiltonian $\mathcal{H}_{\text{eff}}$ allows to produce GHZ states by turning on the interaction for a definite duration t_{GHZ}. It was shown in Refs. 22,24 that a Hamiltonian of the type $\tilde{g}\hat{J}_x^2$ will produce a GHZ state after the time $\pi/(2\tilde{g})$, starting for instance from the state $\bigotimes_q |1\rangle_q$. Implementation of such scheme in other qubit system has also been proposed[32,33]. The multi-qubit gate $\exp(i\pi\hat{J}_x^2/2)$ is sometimes referred to as Mølmer-Sørensen gate.

We conveniently choose an initial state with all the qubits aligned in the same direction, that is the maximal superposition state $|\psi_0\rangle = \bigotimes_q (|0\rangle_q + |1\rangle_q)/\sqrt{2}$. We assume that the qubits and the resonator are initially in a product state and the state of the resonator at $t = 0$ is denoted ρ_{res},

$$\rho(t=0) = |\psi_0\rangle\langle\psi_0| \otimes \rho_{\text{res}}\,. \tag{10}$$

Moreover $|\psi_0\rangle = |J_x = N/2\rangle$ and can be expressed as a linear superposition of the states $|J_z\rangle$ (see Appendix),

$$|\psi_0\rangle = \frac{1}{\sqrt{2^N}}\sum_{k=0}^{N}\sqrt{\binom{N}{k}}\,|J_z = k - N/2\rangle\,. \tag{11}$$

We define $\rho(t)$ as the density matrix evolving under the action of the time-evolution operator $U(t) = \exp(-i\mathcal{H}_{\text{eff}}t)$, where $\mathcal{H}_{\text{eff}}$ is the effective Hamiltonian Eq. (9),

$$\rho(t) = U(t)\,\rho(t=0)\,U^\dagger(t). \tag{12}$$

We assumed that $g/\Delta \ll 1$ and therefore we have neglected the effect of the canonical transformation e^S on the state vector. This turns out to be particularly useful, since $U(t)$ is diagonal in the basis $|n\rangle$, we can describe directly the time evolution of the reduced density matrix of the qubits $\rho_{\text{qb}}(t)$, obtained by tracing over the resonator state,

$$\begin{aligned}\rho_{\text{qb}}(t) &:= \text{Tr}_{\text{res}}[\rho(t)] \\ &= \frac{1}{2^N}\sum_{n,k,k'}\langle n|\rho_{\text{res}}|n\rangle\sqrt{\binom{N}{k}\binom{N}{k'}}e^{-i(\varphi_{k,n}(t)-\varphi_{k',n}(t))} \\ &\qquad |J_z = k - N/2\rangle\langle J_z = k' - N/2|\,,\end{aligned} \tag{13}$$

where $\varphi_{k,n}(t) = k\,(\Omega t + 2\chi t n + \tilde{g}t(N-k))$.

The Greenberger-Horne-Zeilinger (GHZ) states we aim to produce are of the following form,

$$|\text{GHZ}^\pm\rangle = \frac{1}{\sqrt{2}}\left(\bigotimes_{q=1}^{N}\frac{|0\rangle_q + |1\rangle_q}{\sqrt{2}} \pm i\bigotimes_{q=1}^{N}\frac{|0\rangle_q - |1\rangle_q}{\sqrt{2}}\right), \tag{14}$$

which are standard GHZ states up to single qubit rotations. These states can be expressed as a linear superposition of the states $|J_z\rangle$ as well (see Appendix):

$$|\text{GHZ}^\pm\rangle = \sum_{k=0}^{N}\frac{1 \pm i\,e^{i\pi k}}{\sqrt{2^N}\sqrt{2}}\sqrt{\binom{N}{k}}|J_z = k - N/2\rangle\,. \tag{15}$$

To see why a GHZ state is produced after some time t_{GHZ} we consider the effects of either $\exp(i\tilde{g}t\hat{J}_z^2)$ or $\exp[i\tilde{g}t(\hat{J}_z^2 - \hat{J}_z)]$ (for N either even or odd) on the state $|J_z = k - N/2\rangle$. We establish that one of the two possible GHZ states Eq. (14) is produced when $\tilde{g}t = \pi/2$ by noticing that

$$\frac{1+ie^{i\pi(k+\frac{N}{2}-1)}}{\sqrt{2}} = e^{-i\frac{\pi}{4}+i\frac{\pi}{2}(k-\frac{N}{2})^2}\,, \quad N \text{ even}\,,$$

$$\frac{1+ie^{i\pi(k+\frac{N-1}{2})}}{\sqrt{2}} = e^{-i\frac{\pi}{8}+i\frac{\pi}{2}\left[(k-\frac{N}{2})^2-(k-\frac{N}{2})\right]}\,, \quad N \text{ odd}\,.$$

Thus, a GHZ state is produced for every odd multiple of time $t_{\rm GHZ}$. The shortest preparation time is $t_{\rm GHZ} = \pi/(2\tilde{g})$

However the remaining term of the effective Hamiltonian in Eq. (9), the one which is proportional to $\hat{J}_z$, induces a collective rotation of the final state. The rotation angle depends again on N and the state of the resonator. The state $\rho_{\rm qb}(t_{\rm GHZ})$ is,

$$\rho_{\rm qb}(t_{\rm GHZ}) = \sum_n \langle n|\rho_{\rm res}|n\rangle\, |{\rm GHZ}(\alpha_n)\rangle\langle {\rm GHZ}(\alpha_n)|\,. \quad (16)$$

Here,

$$|{\rm GHZ}(\alpha)\rangle = e^{-i\alpha\hat{J}_z}\frac{1}{\sqrt{2}}\left(\bigotimes_{q=1}^{N}\frac{|0\rangle_q+|1\rangle_q}{\sqrt{2}} + e^{i\pi\frac{N-1}{2}}\bigotimes_{q=1}^{N}\frac{|0\rangle_q-|1\rangle_q}{\sqrt{2}}\right), \quad (17)$$

and $2\alpha_n/\pi = (\Omega + 2n\chi)/\tilde{g}$ for N even. For N odd, $2\alpha_n/\pi = (\Omega + 2n\chi)/\tilde{g} - 1$, and the relative phase $\exp(i\pi(N-1)/2)$ in Eq. (17) is changed to $\exp(i\pi N/2)$.

We notice that the produced states $\rho(t_{\rm GHZ})$ is not exactly the state depicted in Eq. (14) and therefore certain constraints on the angles α_n in Eq. (16) are required to generate the proper state $|{\rm GHZ}^+\rangle$. At low temperature, only the ground state of the resonator is significantly populated and $\langle 0|\rho_{\rm res}|0\rangle \gg \langle n|\rho_{\rm res}|n\rangle$ for $n \geq 1$. Thus we can restrict our considerations to $\alpha_{n=0}$ and this translates to some condition on the ratio $\Omega/\tilde{g}$.

To illustrate this we consider the resonator to be initially in its ground state $\rho_{\rm res} = |n=0\rangle\langle n=0|$. The state $|{\rm GHZ}^+\rangle$ is indeed produced at $t_{\rm GHZ}$, provided we can tune the frequencies Ω and $\tilde{g}$ such that

$$\frac{\Omega}{\tilde{g}} = 4m + 2 - N\,, \qquad m \in \mathbb{Z}\,. \quad (18)$$

If the above condition cannot be satisfied, some correcting pulse $\exp(i\delta_N\hat{J}_z)$ can be applied to the final state $\rho_{\rm qb}(t_{\rm GHZ})$ to obtain a proper $|{\rm GHZ}^+\rangle$ state. The appropriate pulse length δ_N depends on N and the ratio $\Omega/\tilde{g}$,

$$\delta_N = \frac{\pi}{2}\left[\left(\frac{\Omega}{\tilde{g}} + N - 2\right) \bmod 4\right]. \quad (19)$$

Furthermore $\delta_N = 0$ implies Eq. (18).

If not only the ground state of the resonator is populated, higher photon numbers n produce rotated GHZ states, according to Eq. (16). We notice that $\langle {\rm GHZ}(\alpha_n)|{\rm GHZ}(\alpha_{n+k})\rangle = \cos^N(k\pi\chi/(2\tilde{g}))$, which means that if a $|{\rm GHZ}^+\rangle$ state is produced for excitation number n, a slightly rotated state $\exp(-i\pi\chi\hat{J}_z/\tilde{g})|{\rm GHZ}^+\rangle$ is produced for $n+1$ (since $\chi < \tilde{g}$). Assuming some correcting pulse $\exp(i\delta_N\hat{J}_z)$ has been applied, the reduced density matrix of the qubits $\rho_{\rm qb}$ is a mixture of rotated GHZ states with classical probabilities depending only on the initial state of the resonator,

$$\begin{aligned} &e^{i\delta_N\hat{J}_z}\rho_{\rm qb}(t_{\rm GHZ})e^{-i\delta_N\hat{J}_z} \\ &= \langle 0|\rho_{\rm res}|0\rangle|{\rm GHZ}^+\rangle\langle{\rm GHZ}^+| \\ &+ \sum_{n>0}\langle n|\rho_{\rm res}|n\rangle e^{-i\pi n\frac{\chi}{\tilde{g}}\hat{J}_z}|{\rm GHZ}^+\rangle\langle{\rm GHZ}^+|e^{i\pi n\frac{\chi}{\tilde{g}}\hat{J}_z}\,. \end{aligned} \quad (20)$$

We will now show that it is possible to choose realistic physical parameters in agreement with our assumptions. Transmon qubits have typical frequencies $\Omega/2\pi$ around 10 GHz and coplanar waveguide resonators (the quantum bus) can be realized with frequencies $\omega/2\pi$ of the order of 10 GHz with high quality factors[30]. Transmon-resonator coupling frequencies $g/2\pi$ around 200 MHz is a reasonable assumption. Detuning the qubits from the resonator such that $g/\Delta \simeq 1/10$ would lead to an effective qubit-qubit coupling of the order of $\tilde{g} = g/10$ and to preparation time $t_{\rm GHZ}$ of approximately 12.5 ns.

IV. MEASURING THE GENERATED GHZ STATES

The question of detecting and probing the states generated in our scheme naturally arises. For $N \geq 4$, there is no unique way to quantify entanglement. We will focus on a measurement of the Bell-Mermin operator[34] defined as

$$\begin{aligned} B &= \frac{e^{i\pi N}}{2i}\left[\bigotimes_{q=1}^{N}\left(\sigma_z^{(q)} - i\sigma_y^{(q)}\right) - \bigotimes_{q=1}^{N}\left(\sigma_z^{(q)} + i\sigma_y^{(q)}\right)\right] \\ &= 2^{N-1}\left(|{\rm GHZ}^+\rangle\langle{\rm GHZ}^+| - |{\rm GHZ}^-\rangle\langle{\rm GHZ}^-|\right)\,, \end{aligned} \quad (21)$$

whose expectation value for N-qubit quantum states is bounded by $|\langle B\rangle| \leq 2^{N-1}$, and the extremal values $\pm 2^{N-1}$ are reached for the states $|{\rm GHZ}^\pm\rangle$. The maximal value predicted by local hidden-variable theory is $\sqrt{2^N}$ ($\sqrt{2^{N-1}}$) for N even (odd), leading to an exponentially increasing violation for the states $|{\rm GHZ}^\pm\rangle$ with N, the number of qubits. Therefore, a measurement of the Bell-Mermin operator leading to a result greater than $\sqrt{2^N}$ ($\sqrt{2^{N-1}}$) indicates the non-local nature of the generated quantum states.

Other bounds can be derived for this operator: e.g., any separable state ρ^S satisfies $|{\rm Tr}(\rho^S B)| \leq 1$. A significant bound can also be derived if the state is m-separable, i.e. describes a system which is partitioned in m subsystems that only share classical correlations. In other words, a *pure* state is called m-separable, for $1 < m \leq N$, if it can be written as a product of m states,

$$|\psi^m\rangle = \bigotimes_{i=1}^{m} |\psi_i\rangle_{P_i}\,, \tag{22}$$

where the $\{P_i\}$ describe a partition of the N qubits. Thus, a separable state in the traditional sense is N-separable. A *mixed* m-separable state ρ^m is defined as a convex sum of pure m-separable states, which might belong to different partitions[35]. Such an m-separable state satisfies $\mathrm{Tr}(\rho^m B) \leq 2^{N-m}$. Thus, any measurement of B with outcome above 2^{N-2} indicates that the state is not even biseparable (*2-separable*) and demonstrates the existence of genuine N-partite entanglement.

The Bell-Mermin operator expectation value can in principle be obtained experimentally. This operator can be expressed as a sum of parity operators, and inferring its expectation value would require 2^{N-1} parity measurements,

$$\langle B\rangle = \sum_{l=1\ \mathrm{odd}}^{N} \sum_{p} (-1)^{N-\frac{l+1}{2}} \left\langle \bigotimes_{q=1}^{N-l} \sigma_z^{p(q)} \bigotimes_{q'=N-l+1}^{N} \sigma_y^{p(q')} \right\rangle. \tag{23}$$

For each term, l is the number of factors σ_y and $\sum_p$ stands for the sum over the $\binom{N}{l}$ permutations p that give distinct products. The states $|\mathrm{GHZ}^{\pm}\rangle$ defined in Eq. (14) are those that give exactly ± 1 for each of the 2^{N-1} terms.

There are therefore 2^{N-1} parity measurements to realize which is possible only if one is able to generate GHZ states with high accuracy in a repeated way. Following Ref. 20, these parity operators could be measured by dispersive readout. Since the frequency of the resonator is ac-Stark shifted $\omega \to \omega + \chi \sum_q \sigma_z^{(q)}$, it is possible to access the value of the operator $\hat{J}_z$. The value of the parity operator $\bigotimes_q \sigma_z^{(q)}$ can then be unambiguously deduced from $J_z = \langle \hat{J}_z\rangle$,

$$\left\langle \bigotimes_{q=1}^{N} \sigma_z^{(q)} \right\rangle = (-1)^{\frac{N}{2}-J_z}\,. \tag{24}$$

Hence, we can measure all the needed parities by rotating the operators $\sigma_y^{(q)}$ appearing in Eq. (23) to $\sigma_z^{(q)}$ using single-qubit rotations.

By means of Eq. (13), we can give an expression for the time evolution of the expectation value of the Bell-Mermin operator, $\langle B(t)\rangle = \mathrm{Tr}\,[B\rho_{\mathrm{qb}}(t)]$. For this purpose we can express the matrix elements of B in the basis of the states $|J_z\rangle$, which diagonalizes the effective Hamiltonian,

$$B = \sum_{k,k'=0}^{N} b_{k,k'}\ |J_z{=}k'{-}N/2\rangle\langle J_z{=}k{-}N/2|\,, \tag{25}$$

where

$$b_{k,k'} = \frac{1}{2i}\sqrt{\binom{N}{k}\binom{N}{k'}}\left[(-1)^k - (-1)^{k'}\right]. \tag{26}$$

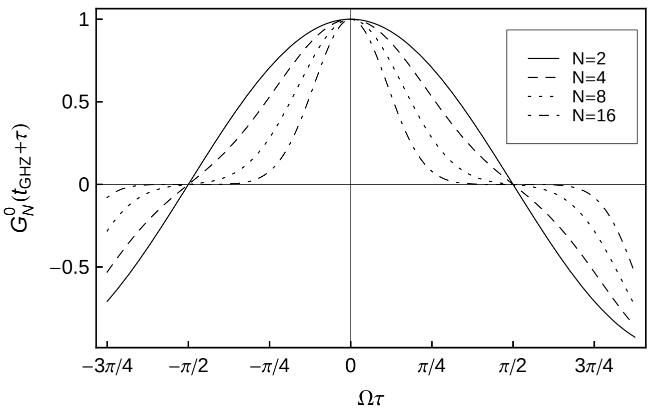

FIG. 2: Behavior of the function $G_N^0(t_{\mathrm{GHZ}}+\tau)$ for different N, assuming for simplicity that $\delta_N = 0$.

Hence, $\langle B(t)\rangle$ can be expressed as a sum of oscillating functions G_N^n, indexed by the photon number n,

$$\langle B(t)\rangle = 2^{N-1}\sum_{n=0}^{\infty}\langle n|\rho_{\mathrm{res}}|n\rangle\ G_N^n(t)\,. \tag{27}$$

The functions G_N^n are Fourier series over a finite range of frequencies $\tilde{\omega}_{k,k'}^n$ defined as $\tilde{\omega}_{k,k'}^n = (k-k')\,[(k+k'-N)\tilde{g} - \Omega - 2n\chi]$,

$$G_N^n(t) = \sum_{k,k'=0}^{N} a_{k,k'}\sin(\tilde{\omega}_{k,k'}^n t)\,, \tag{28}$$

where

$$a_{k,k'} = 2^{-2N}\binom{N}{k}\binom{N}{k'}\left[(-1)^k - (-1)^{k'}\right]. \tag{29}$$

Equation (27) shows that $\langle B(t)\rangle$ is characterized by many oscillations on timescales $\sim t_{\mathrm{GHZ}}$, since the $\tilde{\omega}_{k,k'}^n$ are of the same order as $\Omega \gg \tilde{g}, \chi$. However, the envelope indeed reaches its maximum at t_{GHZ}, provided that only the ground state of the resonator is significantly populated. These fast oscillations are the manifestation of local rotations of the qubits, Eqs. (16-17). We have seen that this issue can be solved equivalently in two different ways and that the state $|\mathrm{GHZ}^+\rangle$ is indeed generated after t_{GHZ}, either by applying some correcting pulse $\exp(i\delta_N \hat{J}_z)$, defined in Eq. (19), or by tuning the frequencies Ω and $\tilde{g}$ to satisfy the condition Eq. (18). Assuming for simplicity that $\delta_N = 0$, we have then

$$G_N^n(t_{\mathrm{GHZ}}) = \cos^{2N}\left(n\frac{\pi}{2}\frac{\chi}{\tilde{g}}\right) - \sin^{2N}\left(n\frac{\pi}{2}\frac{\chi}{\tilde{g}}\right). \tag{30}$$

The fast oscillations of $\langle B(t)\rangle$ around t_{GHZ} become sharper as the number of qubits N increases, as shown in Fig. 2. In the simpler case $\delta_N = 0$, the behavior of G_N^0 around t_{GHZ} is given by

$$G_N^0(t_{\mathrm{GHZ}}+\tau) \simeq 1 - \tau^2\frac{N\Omega^2}{4}, \quad |\tau| \ll \frac{1}{\Omega}\,, \tag{31}$$

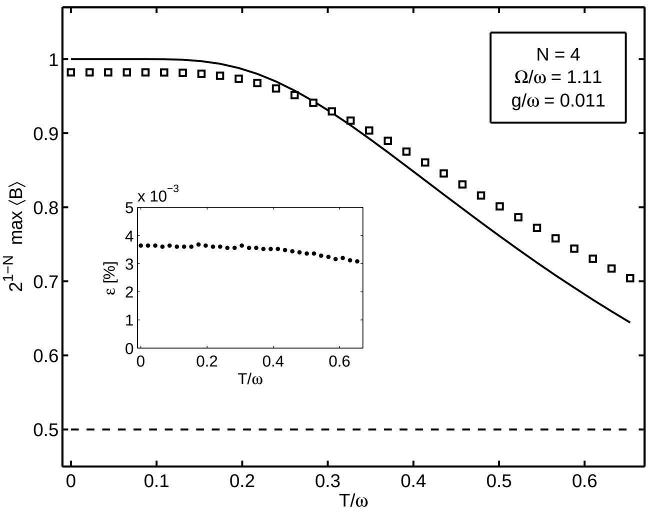

FIG. 3: Temperature dependence of the maximum $\max\langle B\rangle$ of $\langle B(t)\rangle$, for $t \sim t_{\rm GHZ}$ (squares), normalized by 2^{N-1}. The solid line shows the theoretical bound $\tanh(\omega/(2T))$ for a resonator initially in the thermal state Eq. (33). Inset: relative deviation $\varepsilon = t_{\rm max}/t_{\rm GHZ} - 1$ of the time $t_{\rm max}$ at which $\max\langle B\rangle$ is realized compared to the predicted time $t_{\rm GHZ} = \pi/(2\tilde{g})$. Here we considered $N = 4$ qubits and the parameters are $\Omega/\omega = 1.105$, $g/\omega = 0.0105$ and thus $g/\Delta \simeq 0.1$. Local hidden-variable theory only allows values of $\langle B\rangle$ below the dashed line. For $N = 4$ this value also corresponds to the upper bound for biseparable states.

and that also means that we need a higher precision, for larger N, in controlling either the protocol time $t_{\rm GHZ}$ or the correcting pulse.

Finally, the maximal value $\langle B(t_{\rm GHZ})\rangle$ can reach depends only on the initial state of the resonator $\rho_{\rm res}$, provided the above considerations have been taken into account. Equations (20) and (21) show that

$$2^{1-N}{\rm Tr}\left[B(e^{i\delta_N \hat{J}_z}\rho_{\rm qb}(t_{\rm GHZ})e^{-i\delta_N \hat{J}_z})\right] = \sum_{n=0}^{\infty}\langle n|\rho_{\rm res}|n\rangle\left[\cos^{2N}\left(n\frac{\pi}{2}\frac{\chi}{\tilde{g}}\right) - \sin^{2N}\left(n\frac{\pi}{2}\frac{\chi}{\tilde{g}}\right)\right]. \quad (32)$$

For instance, we assume $\rho_{\rm res}$ to be a thermal state characterized by a temperature T,

$$\rho_{\rm res} = \left(1 - e^{-\omega/T}\right)\sum_n e^{-n\omega/T}|n\rangle\langle n|. \quad (33)$$

In this simple case, the outcome of the Bell-Mermin operator measurement $\langle B(t_{\rm GHZ})\rangle$ should be at least $2^{N-1}\tanh(\omega/(2T))$.

A numerical evaluation of $\langle B(t)\rangle$, using the Jaynes-Cummings Hamiltonian Eq. (3), shows good agreement with our theoretical analysis. We consider the ideal case of homogeneous qubit and coupling frequencies and we choose frequencies satisfying Eq. (18) such that $\delta_N = 0$. We look for the maximal value of $\langle B(t)\rangle$ around $t_{\rm GHZ}$, that is for $|t - t_{\rm GHZ}| < \frac{\pi}{2\Omega}$, and for the time $t_{\rm max}$ at which this maximal value is realized. The results for $N = 4$ qubits are shown in Fig. 3.

V. INHOMOGENEOUS COUPLING FREQUENCIES

To estimate whether our scheme is robust against small random deviations in the physical parameters, we consider small inhomogeneities in the coupling strengths $g_j^{(q)}$. This effect will be investigated numerically and, for this purpose we compute the real-time evolution of the Bell-Mermin operator, using the Jaynes-Cummings Hamiltonian Eq. (3), truncated to the two lowest levels of the transmon qubits. This should capture the main features of this effect, since in our effective description of the system Eq. (8), the third levels of the transmon qubits only affect the ac-Stark shifts $\chi^{(q)}$ and renormalize the resonator frequency. Assuming the qubit transition frequencies are still homogeneous $\omega_{01}^{(q)} = \Omega$, the inhomogeneity of the coupling frequencies $g_0^{(q)}$ produces inhomogeneous qubit-qubit couplings coefficients $\tilde{g}_{00}^{(qq')} = |g_0^{(q)}g_0^{(q')}|/\Delta$.

The coupling constants $g_0^{(q)}$ are assumed to be Gaussian distributed with mean g and standard deviation δg. The notation $\{g_q\}$ denotes a particular set of coupling frequencies $g_0^{(q)}$. The real-time evolution of the Bell-Mermin operator for one set of coupling frequencies $\{g_q\}$ is denoted $\langle B_{\{g_q\}}(t)\rangle$.

For a given number n_r of random realizations $\{g_q\}$ (n_r around 200) with fixed δg, we first calculate the mean value,

$$\langle\bar{B}(t)\rangle = \frac{1}{n_r}\sum_{\{g_q\}}\langle B_{\{g_q\}}(t)\rangle. \quad (34)$$

Then, the maximal value $\langle\bar{B}(t_{\rm max})\rangle$ defined by

$$\langle\bar{B}(t_{\rm max})\rangle = \max_{t\geq 0}\langle\bar{B}(t)\rangle \quad (35)$$

is found. Finally the variances, above and below the maximal mean value $\langle\bar{B}(t_{\rm max})\rangle$, of the particular set $\left\{\langle B_{\{g_q\}}(t_{\rm max})\rangle\right\}$ are calculated. The variances are calculated separately above and below, because the $\langle B_{\{g_q\}}(t_{\rm max})\rangle$ are not Gaussian distributed. We also calculate the median among the $\langle B_{\{g_q\}}(t_{\rm max})\rangle$ and notice that the distribution is strongly asymmetric.

Results for $N = 4$ and $\delta g/g$ between 0 to 20 % are shown in Fig. 4. The time at which the maximum is attained is generally in good agreement with the predicted value $t_{\rm GHZ} = \pi/(2\tilde{g})$, as long as g/Δ is small. The value of $\langle\bar{B}(t_{\rm max})\rangle$ remains close to the ideal one for $\delta g/g$ of the order of a few percents and thus we notice that our scheme can tolerate some inhomogeneity in the coupling constants.

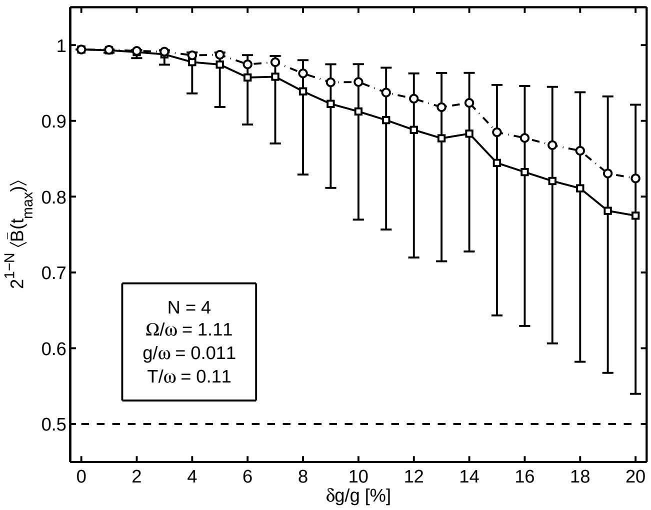

FIG. 4: Effect of inhomogeneous coupling frequencies $g_0^{(q)}$ with mean g and standard deviation δg. We show the dependence of the maximal mean value $\langle \bar{B}(t_{\max})\rangle$ of $\langle B_{\{g_q\}}(t)\rangle$ on $\delta g/g$ for $t \sim t_{\rm GHZ}$ (squares). The error bars show the standard deviation of $\langle B_{\{g_q\}}(t_{\max})\rangle$ above and below the mean value. The median of $\langle B_{\{g_q\}}(t_{\max})\rangle$ (circles) is clearly above the mean value. Local hidden-variable theory only allows values of $\langle B\rangle$ below the dashed line. For $N = 4$ this value also corresponds to the upper bound for biseparable states.

VI. CONCLUSION

To conclude, we have shown that it is possible to generate multipartite Greenberger-Horne-Zeilinger states on a set of transmon qubits in a circuit QED setup in a one-step deterministic protocol. In the dispersive limit $g \ll \Delta$, such a system behaves as a fully connected qubit network with exchange interactions proportional to $\tilde{g} = g^2/\Delta$. The preparation time of the protocol is inversely proportional to $\tilde{g}$. The non-local nature of the generated state can be investigated using a Bell-Mermin inequality. Moreover, we have derived and applied bounds on the expectation value of the Bell-Mermin operator as a detection criterion for genuine N-partite entanglement. Finally we have shown that our scheme is robust against small inhomogeneities in the coupling frequencies. The implementation proposed here looks like a promising way to generate GHZ states, and hopefully can be experimentally realized in a circuit QED setup.

VII. ACKNOWLEDGMENT

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Appendix: Schwinger representation of total spin operators

We present briefly the Schwinger representation[36] of the total spin operators $\hat{J}_{x,y,z} = \frac{1}{2}\sum_q \sigma_{x,y,z}^{(q)}$. This turns out to be particularly useful for calculations in the subspace of $\hat{J}^2$-eigenstates with maximal eigenvalue $\frac{N}{2}\left(\frac{N}{2}+1\right)$ where N is the number of spins. From now on we set $J = N/2$ and denote the states $|J = N/2, J_{x,y,z}\rangle$ by $|J_{x,y,z}\rangle$.

States like $|J_z\rangle$ are sometimes referred to as Dicke states[37], they form a complete basis of symmetric N-qubit states, i.e., states invariant under any permutation of qubits. We use for each qubit the standard basis $\{|0\rangle, |1\rangle\}$ with the convention $\sigma_z^{(q)}|1\rangle_q = |1\rangle_q$ and $\sigma_z^{(q)}|0\rangle_q = -|0\rangle_q$,

$$\begin{aligned} &|J_z = k - N/2\rangle \\ &= \frac{1}{\sqrt{\binom{N}{k}}}\sum_p |1\rangle_{p(1)}\cdots|1\rangle_{p(k)}|0\rangle_{p(k+1)}\cdots|0\rangle_{p(N)}\,, \end{aligned} \tag{A.1}$$

with $0 \le k \le N$ and where the sum is taken over the $\binom{N}{k} = \frac{N!}{k!(N-k)!}$ nonequivalent possible permutations p that give different product states.

The operators $\hat{J}_i$ are defined by means of two independent bosonic operators a and b, with commutation relations $[a, a^\dagger] = [b, b^\dagger] = 1$ and $[a, b] = [a, b^\dagger] = 0$,

$$\begin{aligned} \hat{J}_x &= \frac{1}{2}(b^\dagger a + a^\dagger b)\,, \\ \hat{J}_y &= \frac{1}{2i}(b^\dagger a - a^\dagger b)\,, \\ \hat{J}_z &= \frac{1}{2}(b^\dagger b - a^\dagger a)\,, \end{aligned} \tag{A.2}$$

fulfilling the SU(2) algebra $[\hat{J}_l, \hat{J}_m] = i\epsilon_{lmn}\hat{J}_n$. Eigenstates of $\hat{J}_z$ can be expressed as

$$|J, J_z\rangle = \frac{\left(b^\dagger\right)^{J+J_z} \mid \left(a^\dagger\right)^{J-J_z}}{\sqrt{(J+J_z)!(J-J_z)!}}|n_a = 0, n_b = 0\rangle\,, \tag{A.3}$$

where $|n_a = 0, n_b = 0\rangle$ is the vacuum state of the operators a and b. Since the choice of the operators a and b is not unique, we can equivalently introduce the operators $c = (a - b)/\sqrt{2}$ and $d = (a + b)/\sqrt{2}$, leading to $\hat{J}_x = \frac{1}{2}(d^\dagger d - c^\dagger c)$ and

$$|J, J_x\rangle = \frac{\left(d^\dagger\right)^{J+J_x} \mid \left(c^\dagger\right)^{J-J_x}}{\sqrt{(J+J_x)!(J-J_x)!}}|n_a = 0, n_b = 0\rangle\,. \tag{A.4}$$

We straightforwardly obtain the decomposition of the states $|J,J_x\rangle$ in terms of $|J,J_z\rangle$ and in particular

$$\begin{aligned} |J_x\!=\!\pm N/2\rangle &= \bigotimes_{q=1}^{N} \frac{|0\rangle_q \pm |1\rangle_q}{\sqrt{2}} \\ &= \frac{\left(a^\dagger \pm b^\dagger\right)^N}{\sqrt{2^N N!}} |n_a\!=\!0, n_b\!=\!0\rangle \\ &= \frac{1}{2^{N/2}} \sum_{k=0}^{N} (\pm 1)^k \sqrt{\binom{N}{k}} |J_z\!=\!k\!-\!N/2\rangle\,. \end{aligned} \tag{A.5}$$

Defining the ladder operators $\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$ of the total spins, the Dicke states can also be expressed as

$$\begin{aligned} |J_z\!=\!k\!-\!N/2\rangle &= \frac{\left(\hat{J}_+\right)^k}{k!\sqrt{\binom{N}{k}}} \bigotimes_{q=1}^{N} |0\rangle_q \\ &= \frac{\left(\hat{J}_-\right)^{N-k}}{(N-k)!\sqrt{\binom{N}{k}}} \bigotimes_{q=1}^{N} |1\rangle_q\,. \end{aligned} \tag{A.6}$$

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