

# Cooling a micro-mechanical resonator by quantum back-action from a noisy qubit

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We study the role of qubit dephasing in cooling a mechanical resonator by quantum back-action. With a superconducting flux qubit as a specific example, we show that ground-state cooling of a mechanical resonator can only be realized if the qubit dephasing rate is sufficiently low.

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A micro (nano) mechanical resonator (MR) can be cooled down through quantum back-action of coupled auxiliary mesoscopic systems. Environmental fluctuations induce both relaxation and dephasing to the auxiliary systems. Previous studies [1–11] show that relaxation plays an essential role to dissipate the MR energy to the environment. In this paper, we address the function of environment-induced dephasing in the back-action cooling.

Preparing quantum systems in their ground states is one way to initialize them for the implementation of many quantum protocols. If the MR is to be cooled to milli-Kelvin temperatures, novel cooling techniques other than dilution refrigeration are required. Experimentally, cooling by an optical cavity (e.g. [12–14] and references therein) and a superconducting single-electron transistor (SSET) has been demonstrated [15], but both techniques are still far from reaching the quantum ground state of the MR. Theoretical proposals have predicted the possibility of ground-state cooling of the MR [1–3, 7–9]. The basic idea of these proposals resembles laser cooling of trapped ions. The dissipative auxiliary system (such as the internal levels of an ion, the optical cavity, the SSET, the quantum dot, etc.) acts as a structured bath of the system to be cooled (in our case, the MR). The relaxation and excitation rate of the auxiliary system obeys the detailed balance relation determined by the external bath temperature. Introducing a red-detuned drive modifies the dissipative nature of the auxiliary system: absorption processes in the auxiliary system and associated MR phonon emission processes dominate over the inverse processes, so that detailed balance is broken. These phonon emission processes extract energy from the MR and dissipate it to the external bath through the subsequent spontaneous emission of the auxiliary system energy quanta. Therefore, the limit of the cooling procedure is determined primarily by: (1) the environmental noise of the auxiliary system and (2) the way the dissipative nature of the auxiliary system is modified by the drive.

The environmental noise acting on the auxiliary system leads to both relaxation and dephasing processes. The

cooling efficiency of trapped-ion or optomechanical cooling is only influenced by the relaxation process. In the case of cooling by driving a solid-state qubit [5, 7, 8, 16], pure dephasing can usually be neglected by biasing the qubit at (or close to) the degeneracy point where the pure dephasing rate is negligibly small. However, the first-order photon excitation by absorbing the energy from the low-frequency phonon and the linear drive vanishes at this point, only a small second-order photon interaction remains [7]. Thus, the cooling becomes less efficient. This motivates us to bias the qubit away from the degeneracy point and take the qubit dephasing into account. This consideration is especially important for solid-state qubits because the dephasing rate is much larger than the relaxation rate away from the degeneracy point and it is more difficult to suppress dephasing since it is associated with low-frequency noise.

In this paper, we use the master-equation approach to investigate the ground-state cooling of the MR by a flux qubit. The influence of both qubit dephasing and relaxation is studied. Assuming a  $1/f$  noise spectrum for the flux qubit, we show that ground-state cooling of the mechanical resonator is possible under current experimental conditions.

The system we consider is a micro-mechanical beam interacting with a superconducting flux qubit [17, 18], see Fig. 1. In the  $x$ - $y$  plane, a doubly-clamped micro-mechanical beam with an effective length  $L_0$  is incorporated in a superconducting loop with three small-capacitance Josephson junctions. This mechanical beam can be created from an MBE-grown heterostructure coated with superconducting material [19], or a self-supporting metallic air bridge. The fundamental vibration mode of the beam can be well approximated as a harmonic resonator with oscillation frequency  $\omega_b$ . With a proper bias magnetic flux, two classically stable states of the 3-Josephson-junction loop carry persistent currents in opposite directions. There is a finite tunneling rate  $\Delta$  between the two classical persistent-current states (throughout this article, we let  $\hbar = 1$ ). This two-level subspace is far below the other energy levels and forms a

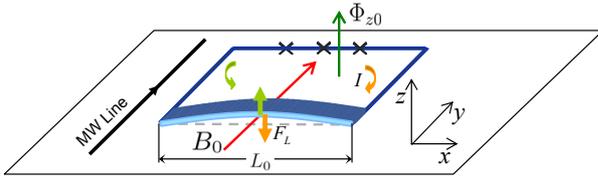


FIG. 1: (Color online) Schematic diagram of our setup. A doubly-clamped mechanical beam is incorporated in a superconducting flux qubit (Josephson junctions are indicated by crosses). The initial bias in the loop is controlled by a magnetic flux  $\Phi_{z0}$  in the  $z$ -direction. A coupling magnetic field  $B_0$  is applied in the  $y$ -direction, leading to a coupling of the motion of the beam and the qubit because the supercurrent leads to a Lorentz force on the beam. A microwave (MW) line introduces a microwave bias to the qubit loop.

flux qubit [20, 21]. The qubit ground state  $|g\rangle$  and excited state  $|e\rangle$  are coherent superpositions of two persistent current states denoted by  $|0\rangle$  (clockwise current state) and  $|1\rangle$  (counter-clockwise current state). The energy spacing between the two eigenstates is  $\Omega = \sqrt{\Delta^2 + \varepsilon_0^2}$  where  $\varepsilon_0 = 2I_p(\Phi_{\text{ext}} - \Phi_0/2)$  is the energy spacing of the two classical current states ( $|0\rangle$  and  $|1\rangle$ ), with  $\Phi_{\text{ext}}$  the external magnetic flux through the loop,  $I_p$  the maximum persistent current in the loop, and  $\Phi_0 = h/(2e)$  the flux quantum. A microwave line is placed close to the circuit and can be used as a microwave drive with frequency  $\omega_d$  acting on the flux qubit. The qubit Hamiltonian is written as

$$H_q = (\Delta/2)\sigma_x + (\varepsilon_0/2)\sigma_z + A\sigma_z \cos(\omega_d t) \quad (1)$$

where  $A$  characterizes the amplitude of the microwave drive and  $\sigma_x \equiv |1\rangle\langle 0| + |0\rangle\langle 1|$ ,  $\sigma_z \equiv |0\rangle\langle 0| - |1\rangle\langle 1|$ . We assume that the drive is near resonant with the qubit  $|\delta\omega|/(\omega_d + \Omega) \ll 1$  (here,  $\delta\omega = \Omega - \omega_d$  is the detuning between the qubit free energy and the drive), and that the drive amplitude satisfies  $A/\omega_d \ll 1$ . In the presence of a magnetic field  $B_0$  along the  $y$ -direction, the supercurrent generates a Lorentz force  $F_L$  on the MR along the  $z$ -direction. This force couples the flux qubit with the motion of the MR. The coupling Hamiltonian is  $g(a + a^\dagger)\sigma_x$  with  $g = B_0 I_p L_0 \delta_0$ . Here,  $\delta_0 = \sqrt{1/(2m\omega_b)}$  is the harmonic oscillator length (mean-square zero-point displacement). As shown previously using a semi-classical treatment [16], this configuration can serve as an ‘‘on-chip refrigerator’’ for the MR under a proper drive: the Lorentz force produced by the flux qubit induces a passive back-action on the MR, damping its thermal motion.

At low frequencies, the decoherence of the superconducting flux qubit is dominated by  $1/f$  noise. This noise induces qubit relaxation and dephasing [22–24]. In the absence of the microwave drive, the qubit relaxation (excitation) rates  $\Gamma_\downarrow^{(0)}$ ,  $\Gamma_\uparrow^{(0)}$  satisfy the detailed balance relation  $\Gamma_\downarrow^{(0)}/\Gamma_\uparrow^{(0)} = \exp(\Omega/k_B T_0)$ , where  $T_0$  is the tem-

perature of the external bath. Since  $\Omega \gg k_B T_0$  in our case, we neglect  $\Gamma_\uparrow^{(0)}$  in this discussion. The qubit pure dephasing rate  $\Gamma_\varphi^{(0)}$  is almost proportional to the qubit bias [22, 23] in the vicinity of the degeneracy point so that

$$\Gamma_\varphi^{(0)} = \alpha \varepsilon_0. \quad (2)$$

In the presence of a near-resonant microwave drive, in the rotating frame of frequency  $\omega_d$ , there are new eigenstates which are superpositions of the initial eigenstates  $|e\rangle$  and  $|g\rangle$ . The new eigenstates are the qubit-microwave dressed states [25]. The dephasing of the undressed states also contributes to the relaxation and excitation of the dressed states. The relaxation ( $\Gamma_\downarrow$ ), the excitation ( $\Gamma_\uparrow$ ) and the dephasing rate ( $\Gamma_\varphi$ ) of the dressed states can be generally written as

$$\Gamma_{\downarrow(\uparrow, \varphi)} \equiv \Gamma_{\downarrow(\uparrow, \varphi)R} + \Gamma_{\downarrow(\uparrow, \varphi)D}, \quad (3)$$

where  $\Gamma_{\dots R}$  denotes the contribution from the relaxation process and  $\Gamma_{\dots D}$  from the dephasing process without drive. The derivation of the Lindblad-form master equation of the driven qubit in the rotating frame leads to the following explicit expressions for the relaxation (excitation) rates:

$$\Gamma_{\downarrow(\uparrow)R} = \frac{\Gamma_\downarrow^{(0)}(1 + \cos\phi)^2 \coth(\beta(\omega_0 + \omega_d)/2) \pm 1}{4 \coth(\beta\Omega/2) + 1} + \frac{\Gamma_\downarrow^{(0)}(1 - \cos\phi)^2 \coth(\beta(\omega_0 - \omega_d)/2) \mp 1}{4 \coth(\beta\Omega/2) + 1} \quad (4)$$

$$\Gamma_{\downarrow(\uparrow)D} = \frac{\Gamma_\varphi^{(0)} \sin^2\phi \coth(\beta\omega_0/2) \pm 1}{2(1 + \coth(\beta\omega_c/2))}, \quad (5)$$

$$\Gamma_{\varphi R} = \frac{\Gamma_\downarrow^{(0)} \sin^2\phi}{\coth(\beta\Omega/2) + 1} \quad (6)$$

$$\Gamma_{\varphi D} = \Gamma_\varphi^{(0)} \cos^2\phi \quad (7)$$

Equations (4)-(7) are obtained by assuming a  $1/f$  noise spectrum [26]. Here,  $\beta = 1/(k_B T_0)$ ,  $\sin\phi = A'/\omega_0$ ,  $\cos\phi = \delta\omega/\omega_0$ ,  $\omega_0 = \sqrt{\delta\omega^2 + A'^2}$ , and  $A' = -A \sin\theta$  with  $\sin\theta = \Delta/\Omega$  and  $\cos\theta = \varepsilon_0/\Omega$ . Below the cutoff frequency  $\omega_c$  of the  $1/f$  spectrum, we assume the noise spectrum to be white noise.

In the limit of weak qubit-resonator coupling, the Born approximation can be applied and one can trace out the qubit degree of freedom in the composite system master equation. This yields the following equation of motion for the phonon number

$$\dot{n} = -n(\gamma_m + \gamma_q) + (\gamma_m N + \gamma_q N_q). \quad (8)$$

Its solution reads

$$n(t) = n_0 e^{-(\gamma_m + \gamma_q)t} + \frac{\gamma_m N + \gamma_q N_q}{\gamma_m + \gamma_q} \left(1 - e^{-(\gamma_m + \gamma_q)t}\right) \quad (9)$$

and the stable state solution at  $t \gg 1/(\gamma_m + \gamma_q)$  is

$$n = \frac{\gamma_m N + \gamma_q N_q}{\gamma_m + \gamma_q}. \quad (10)$$

Here,  $\gamma_m$  is the damping rate and  $N = [\exp(\beta\omega_b) - 1]^{-1}$  the mean phonon number determined by the thermal bath, whereas  $\gamma_q = g^2[S_{zz}(\omega_b) - S_{zz}(-\omega_b)]$  is the damping rate and  $N_q = S_{zz}(-\omega_b) / (S_{zz}(\omega_b) - S_{zz}(-\omega_b))$  the mean phonon number determined by the qubit bath. Thus, the MR is effectively in contact with two baths: One is the real (external) thermal bath, and the other one is the structured bath formed by the dissipative qubit under drive (for simplicity, we call this the ‘‘qubit bath’’ in the following). Since the MR is coupled via the  $\sigma_z$ -component of the qubit, the MR decoherence related to the qubit bath is determined by the pair correlation function  $S_{zz}(\omega) = \int dt e^{i\omega t} \langle \sigma_z(t) \sigma_z(0) \rangle$ , where the brackets denote the average over the steady state of the qubit that satisfies  $d\langle \sigma_{x,y,z} \rangle / dt = 0$ . An explicit evaluation of  $S_{zz}(\omega)$  leads to

$$S_{zz}(\omega) = \cos^2 \theta \sin^2 \phi \left( 1 + \frac{\kappa_-}{\kappa_+} \right) \frac{2\kappa_0}{2(\omega + \omega_0)^2 + \kappa_0^2} + \cos^2 \theta \sin^2 \phi \left( 1 - \frac{\kappa_-}{\kappa_+} \right) \frac{2\kappa_0}{4(\omega - \omega_0)^2 + \kappa_0^2}. \quad (11)$$

Here,  $\kappa_0 = \Gamma_\uparrow + \Gamma_\downarrow + 2\Gamma_\varphi$ ,  $\kappa_+ = \Gamma_\uparrow + \Gamma_\downarrow$ , and  $\kappa_- = \Gamma_\uparrow - \Gamma_\downarrow$ . Note that the condition to obtain this reduced master equation is the weak coupling between the MR and the two baths, i.e.  $\gamma_m + \gamma_q \ll \Gamma_\downarrow^{(0)}, \omega_b$ .

In the absence of the drive,  $\gamma_q = 0$ , and Eq. (10) shows that the final occupancy number of the MR is unchanged ( $n = N$ ). This is a natural consequence of the second law of thermodynamics. A slightly detuned drive breaks the thermal equilibrium and changes the final phonon number of the MR. Figure 2 shows the dependence of the cooling efficiency  $\eta \equiv N/n - 1$  on the detuning  $\delta\omega$ . It illustrates that cooling ( $\eta > 0$ ) is induced by a red-detuned ( $\delta\omega > 0$ ) drive: the red-detuned drive accelerates the photon emission process so that the MR energy is dissipated into the environment in the subsequent photon emission. In this way the mean phonon number of the MR is decreased.

As an example, suppose we perform this cooling procedure on a MR of length  $L_0 = 5 \mu\text{m}$  with oscillation frequency  $\omega_b = 10 \text{ MHz}$ , spring constant  $k = 0.01 \text{ N/m}$ , and quality factor  $Q_M = 10^4$ . The MR is coupled with a flux qubit loop by an in-plane magnetic field  $B_0 = 6 \text{ mT}$ . The qubit energy splitting is  $\Delta = 5 \text{ GHz}$  at the degeneracy point and  $I_p = 400 \text{ nA}$ . The coupling strength between the qubit and the MR is about  $10.2 \text{ MHz}$ . The initial environmental temperature is assumed to be  $T_0 = 20 \text{ mK}$ . The qubit relaxation rate  $\Gamma_\downarrow^{(0)} \approx 4 \text{ MHz}$  and the excitation rate  $\Gamma_\uparrow^{(0)} \approx 0$  are almost independent of the qubit bias near the degeneracy point while the pure dephasing rate depends linearly on the qubit bias in this

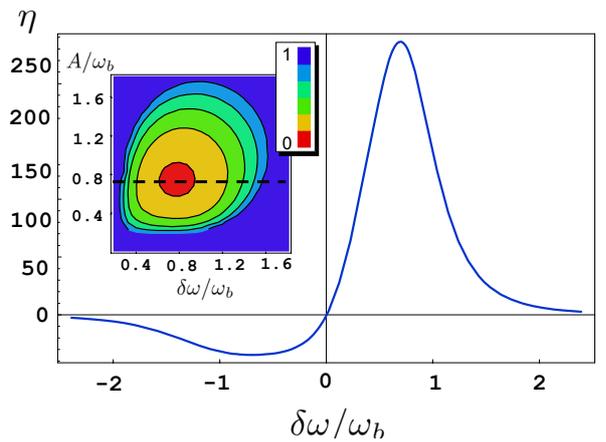


FIG. 2: (Color online) Inset: dependence of the final phonon number  $n$  on the scaled detuning  $\delta\omega/\omega_b$  and the scaled drive amplitude  $A/\omega_b$  (with  $\varepsilon_0 = 800 \text{ MHz}$ ). The plot range is limited from  $n = 0$  to  $n = 1$  (i.e. the ground-state cooling regime). The region with  $n > 1$  is shown in blue. Main panel: cooling power  $\eta \equiv N/n - 1$  as a function of the scaled detuning  $\delta\omega/\omega_b$  between the microwave drive and qubit energy splitting along the dashed line in the inset. The other parameters are the same as those used to estimate the cooling limit in the text.

regime so that  $\Gamma_\varphi^{(0)} \approx 0.008\varepsilon_0$  [22, 23]. The cutoff frequency  $\omega_c$  is assumed to be  $1 \text{ Hz}$ . If the qubit is biased at  $\varepsilon_0 = 800 \text{ MHz}$  and driven by a microwave drive with  $A = 7.4 \text{ MHz}$ ,  $\omega_d = 5.056 \text{ GHz}$ , the effective damping rate and the mean photon number of the qubit bath are  $\gamma_q = 0.4 \text{ MHz}$ ,  $N_q = 0.06$  respectively. The steady state is reached after about  $2.5 \mu\text{s}$ . The phonon number of the MR in the steady state is  $n = 0.14$ . Hence, ground-state cooling can be realized with this setup at the  $1/f$ -noise level characterized by the parameters  $\Gamma_\downarrow^{(0)}$ ,  $\Gamma_\uparrow^{(0)}$ ,  $\Gamma_\varphi^{(0)}$ , and  $\omega_c$  given above which correspond to realistic values. The cooling power can be further improved by increasing the coupling magnetic field  $B_0$ .

In order to achieve ground-state cooling, the power and detuning of the microwave drive should be optimized. As shown in the inset of Fig. 2, the final phonon number  $n$  (or the cooling power  $\eta = N/n - 1$ ) does not depend monotonically on the drive magnitude. This is different from optomechanical cooling where a stronger drive leads to a stronger cooling effect. Qubit-assisted cooling requires a certain resonant condition and the qubit eigenfrequency in the rotating frame is modified by the drive. Previous experiments on cooling a MR by quantum back-action [15] show that the cooling limit is determined by the quantum fluctuations of the auxiliary system. In the following, we analyze how the qubit dephasing and relaxation influences the cooling process in a different way.

Since the decoherence of the qubit is sensitive to the qubit bias, the dependence of the final phonon number on

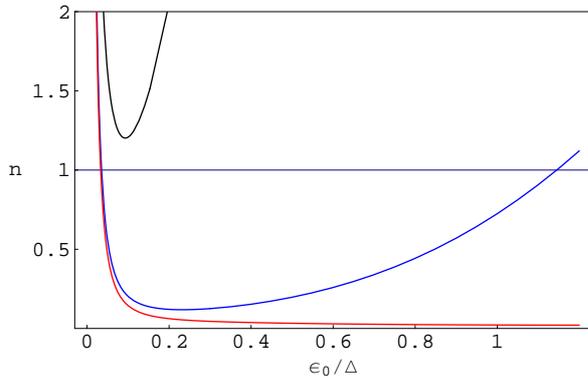


FIG. 3: (Color online) Final phonon number of the MR versus scaled qubit bias. The three curves correspond to qubits with the same relaxation rate ( $\Gamma_{\downarrow}^{(0)} \approx 4$  MHz) but different dephasing rates (from bottom to top  $\alpha = 0$  (red),  $\alpha = 0.008$  (blue), and  $\alpha = 0.08$  (black)).

the qubit bias shown in Fig. 3 reveals the relationship between cooling efficiency and qubit decoherence. Without consideration of dephasing ( $\alpha = 0$ ), the cooling efficiency increases monotonically with the qubit bias. Taking a finite dephasing into account, there is an optimal point for the cooling efficiency as we increase the qubit bias. The cooling efficiency is decreased as the dephasing rate is increased.

The behavior exhibited in Fig. 3 can be understood by making a few simplifications on our previous results. Ground-state cooling requires  $N_q \ll 1$  (or equivalently  $\Gamma_{\uparrow}/\Gamma_{\downarrow} \ll 1$ ) and  $\gamma_q \gg \gamma_m$ . For  $\omega_b \approx \omega_0$ ,  $\Gamma_{\uparrow} \ll \Gamma_{\downarrow}$ , we get  $N_q \approx \Gamma_1^2 / (4(\omega_b + \omega_0)^2)$ , where  $\Gamma_1 = \Gamma_{\uparrow} + \Gamma_{\downarrow} + 2\Gamma_{\varphi}$  is the total dephasing rate. Since  $\Gamma_{\varphi D} \propto \Gamma_{\varphi}^{(0)}$ ,  $N_q$  increases with the increase of pure dephasing. Biasing the qubit to the degeneracy point helps to decrease qubit dephasing and hence decrease  $N_q$ . However, at the degeneracy point,  $\gamma_q = 0$ , photon excitation processes which absorb energy from the low-frequency resonator and the linear drive are not possible any more and the cooling cycle is stopped. Hence, there is an optimal qubit bias point that leads to a lowest final phonon number, see Fig. 3.

The dressed-state relaxation and excitation rates have contributions from the qubit relaxation as well as pure dephasing. Physically, this is because the dressed states of the qubit in the presence of the microwave driving field are superpositions of the qubit eigenstates in the absence of the drive. The qubit pure dephasing contributes significantly to the decoherence process of the dressed qubit and hence modifies the cooling limit. The pure dephasing was not included in the previous studies of the trapped-ion cooling as well as the cooling of a MR by a quantum dot, optical cavity, or SSET. In these cases, the drive modifies the relaxation rate of the auxiliary system but leaves the order of the eigenlevels unchanged. The final

relaxation process is not influenced by the pure dephasing. In the cooling schemes that use a driven charge qubit, dephasing processes are often neglected by biasing the qubit at the degeneracy point. However, as shown in Fig. 3, at the optimal bias point ( $\varepsilon_0 \approx 800$  MHz), the pure dephasing rate  $\Gamma_{\varphi}^{(0)} = 6.4$  MHz is larger than the relaxation rate  $\Gamma_{\downarrow}^{(0)} = 4$  MHz. Hence, it is important to include dephasing in the study of ground-state cooling. Ground-state cooling of a MR can only be realized if the qubit dephasing rate is sufficiently low, see Fig. 3.

In conclusion, we have discussed the quantum theory of the cooling of a MR using the back-action of a superconducting flux qubit. We have shown that the present noise level of the qubit allows ground-state cooling of the MR. The approach used in this paper can be applied to other qubit-assisted back-action cooling schemes to estimate the cooling limit under the influence of pure dephasing. Our method can also be applied to different qubit noise spectra.

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