EUROPHYSICS LETTERS

Europhys. Lett., **53** (2), pp. 251 (2001)

Coherent oscillations in a Cooper-pair box

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(received ; accepted)

PACS. 74.50+r – Proximity effects, weak links, tunneling phenomena, and Josephson effects.
PACS. 73.23Hk – Coulomb blockade; single-electron tunneling.
PACS. 03.67Lx – Quantum computation.

Abstract. – This paper is devoted to an analysis of the experiment by Nakamura *et al.* (Nature **398**, 786 (1999)) on the quantum state control in Josephson junctions devices. By considering the relevant processes involved in the detection of the charge state of the box and a realistic description of the gate pulse we are able to analyze some aspects of the experiment (like the amplitude of the measurement current) in a quantitative way.

The possibility to form coherent superpositions of states is one of the most fundamental properties that distinguishes quantum from classical physics. On the microscopic level, many examples come to mind. Whether it is possible, however, to superpose macroscopically distinct quantum states is controversial and has been debated since the advent of quantum mechanics. Whereas at the beginning it was thought that the macroscopic world is in some sense classical, there have been a number of suggestions over the years on how it might be possible to observe macroscopic quantum coherence in solid-state devices [1]. Superconducting nanocircuits have been used successfully to test quantum mechanics in mesoscopic systems. Examples are the test of the Heisenberg uncertainty principle in a mesoscopic superconductor [2] or experiments on the superposition of charge states in Josephson junctions [3, 4]. More recently, the increasing interest in quantum computation [5] and the search for implementations that can be scaled and integrated has made superconducting circuits promising candidates to realize qubits [6].

In a recent experimental breakthrough, Nakamura *et al.* [7] demonstrated coherent oscillations between two charge states of a superconducting island in a single-electron device, the so-called Cooper-pair box. These states are macroscopically distinct in the sense that they are two different states of an island that contains, say, 10^8 electrons. The superposition is achieved by switching the gate voltage of the box quasi-instantaneously to the point at which

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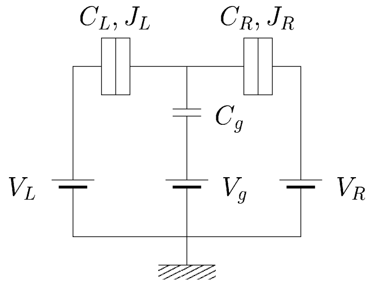

Fig. 1. – Equivalent circuit. The Cooper-pair box corresponds to the configuration $J_L = E_J$, $J_R = 0$, $eV_L = 0$ and $eV_R = -eV \simeq 2\Delta$ using the right lead as a probe gate.

the two charge states, $n = 0$ and $n = 2$, are degenerate. Here n is the number of excess charges on the island (one excess Cooper pair means $n = 2$). As a result, the system performs Rabi oscillations between these two states that are monitored by another weakly coupled tunnel junction.

In this paper, we analyze the experiment by Nakamura *et al.* [7] by solving the appropriate master equation. The reason for performing the work presented here is twofold. Although the description of the coherent oscillations in terms of two-state system is appealing and contains most of the physics there are fundamental questions which are still unanswered. They are related to the mechanisms of decoherence and, particularly important for this experiment, the role of the measuring apparatus and real shape of the gate pulse. Each of these issues leads to computational errors when the Cooper-pair box is used as a (charge)-qubit. Therefore to investigate each of these issues in great detail is a necessary prerequisite for the implementation of a solid state quantum computer with Josephson nanocircuits. The next step in the coherent control of the dynamics of a macroscopic system is the experimental verification of conditional dynamics, i.e. of entanglement. Entangled states, a central concept of quantum mechanics, are difficult to characterize and measure. A quantitative understanding of the single qubit experiment is also relevant for modeling two-qubit gates.

To investigate the results obtained in Ref. [7], we consider the superconducting transistor shown in Fig. 1 and described by the Hamiltonian (see, e.g., [8])

$$H = \frac{(Q+Q_t)^2}{2C} - \sum_{j=L,R} Q_j V_j - \sum_j J_j \cos\phi_j \; . \tag{1}$$

The first two terms define the charging part (H_C), for which we have adopted the effective capacitance model: $C = C_L + C_R + C_g$ with junction capacitance C_L and C_R for the left and right junction and C_g for the gate. When the leads and the gate are biased by voltages V_L, V_R, and V_g, respectively, the total gate-induced charge is given by $Q_t = C_L V_L + C_R V_R + C_g V_g$. Q_j/e is the number of electrons which have passed through the junction j to the central electrode D, and $Q = Q_L + Q_R$ is the total charge on D. The last term in (1) is the Josephson part (H_J) with the Josephson coupling energy J_j and the phase difference ϕ_j across the junctions $j = L, R$. In the Coulomb-blockade regime, where the charging energy $E_C = e^2/2C$ is larger than J_j, the charge Q on the island is quantized in units of electric charge e. Accordingly, we will use the basis of charge states $|n\rangle$ with charge $Q = ne$ where n is integer.

The coupling of the system described by Eq. (1) to the environment leads to decoherence. The environment or bath that we focus on here are the quasiparticles on the two superconducting leads L, R and the central island D

$$H_{qp} = \sum_{\alpha=L,R,D} \sum_{k\sigma} \varepsilon_{k\alpha} \gamma^\dagger_{k\alpha\sigma} \gamma_{k\alpha\sigma} \,. \tag{2}$$

Here, $\gamma^\dagger_{k\alpha}$ ($\gamma_{k\alpha}$) creates (annihilates) a quasiparticle with momentum k and energy $\varepsilon_{k\alpha} = \sqrt{\xi^2_{k\alpha} + \Delta^2}$ in electrode α. Also, ξ_k is the usual single-particle dispersion (with respect to the chemical potential), Δ is the superconducting gap (to simplify the discussion, we assume identical superconductors, as bulk, for all electrodes) and σ labels the spin. The effects of quasiparticle tunneling can be described by the tunneling Hamiltonian

$$H_T = \sum_{j=L,R} \left[e^{-i\phi_j/2} \; X_j + h.c. \right] \,, \tag{3}$$

where $X_j = \sum_{kq\sigma} T_{kq} \gamma^\dagger_{kj\sigma} \gamma_{qD\sigma}$ and T_{kq} is the tunneling amplitude. The total Hamiltonian is given by $H_{tot} = H + H_{qp} + H_T$.

The evolution in time of the reduced density matrix $\rho = Tr_{\gamma^\dagger,\gamma} \rho_{total}$ is governed by the generalized master equation (see, e.g., [9])

$$\begin{aligned} \dot\rho(t) &= -\frac{i}{\hbar} \left[H, \rho(t) \right] \\ &- \sum_j \int_0^\infty ds\; \alpha^>_j(s) \left[e^{-i\phi_j/2}, e^{+i\phi_j(-s)/2} \rho(t) \right] - h.c. \\ &- \sum_j \int_0^\infty ds\; \alpha^<_j(-s) \left[e^{+i\phi_j/2}, e^{-i\phi_j(-s)/2} \rho(t) \right] - h.c. \,. \end{aligned} \tag{4}$$

The correlation functions $\alpha^>_j(t) \equiv \left\langle X_j(t) X^\dagger_j \right\rangle / \hbar^2$ and $\alpha^<_j(t) \equiv \left\langle X^\dagger_j X_j(t) \right\rangle / \hbar^2$ describe the effects of quasiparticle tunneling. More explicitly, the Fourier transforms of these correlation functions can be expressed in terms of the quasiparticle current-voltage characteristics $I^{qp}_j(E)$: $\alpha^>_j(E) = \left[1 + n_B(E)\right] I^{qp}_j(E)/e$ and $\alpha^<_j(E) = n_B(E) I^{qp}_j(E)/e$, where $n_B(E) = 1/\left(e^{\beta E} - 1\right)$ is the thermal distribution function.

The contribution to the transport due to resonant Cooper-pair tunneling has been considered by Averin and Aleshkin [10] and by van den Brink *et al.* [11]. They were interested in the d.c. transport current. In the present paper, we focus on the coherent oscillation between the charge states and its decoherence due to quasiparticle tunneling. In this case, the off-diagonal elements play a crucial role in the dynamics of the reduced density matrix. We note in passing that it is also possible to use H_C instead of H as unperturbed Hamiltonian [10] and the Josephson tunneling H_J in (1) and the quasiparticle tunneling H_T in (3) as perturbations; this is equivalent to our model to second order in the tunneling amplitude.

The Cooper-pair box is probed via quasiparticle tunneling [7]. This can be achieved by configuring the system parameters as follows: the whole system is biased so that Cooper-pair tunneling occurs only across the left junction whereas there is only quasiparticle tunneling across the right junction, $eV_L = 0$ and $eV_R = -eV \sim -2\Delta$. Respectively, we can safely set $J_L = E_J$ and $J_R = 0$, and it is always implied that $k_B T \ll E_J \ll E_C \ll \Delta$. Due to the strong Coulomb repulsion $E_J \ll E_C$, it suffices to consider the subspace of $\{|0\rangle, |1\rangle, |2\rangle\}$. In this basis, the time evolution of the diagonal elements $\sigma_n \equiv \rho_{n,n}$ ($n = 0, 1, 2$) and the off-diagonal element $\chi \equiv \rho_{0,2}$ is given by [10]

$$\dot\sigma_0(t) \;\; = \;\; -i \frac{E_J}{2\hbar} \left[\chi - \chi^* \right] - \Gamma^+(0) \sigma_0(t) + \Gamma^-(1) \sigma_1(t) \tag{5}$$

$$\dot{\sigma}_2(t) = +i\frac{E_J}{2\hbar}\left[\chi - \chi^*\right] - \Gamma^-(2)\sigma_2(t) + \Gamma^+(1)\sigma_1(t) \tag{6}$$

$$\dot{\chi}(t) = +i\frac{E_{2,0}(t)}{\hbar}\chi(t) - i\frac{E_J}{2\hbar}\left[\sigma_0 - \sigma_2\right] - \frac{1}{2}\left[\Gamma^+(0) + \Gamma^-(2)\right]\chi(t)\ , \tag{7}$$

together with the normalization condition $Tr\rho(t) = \sigma_0(t) + \sigma_1(t) + \sigma_2(t) = 1$. Here $E_{2,0} = 4[1 - Q_t(t)/e]E_C$ is the change in the charging energy when a Cooper pair tunnels into the Cooper-pair box across the left junction. In Eqs. (5), $\Gamma^\pm(n) = \Gamma^\pm_L(n) + \Gamma^\pm_R(n)$ and $\Gamma^\pm_j(n)$ is the quasiparticle tunneling rate across the junction $j = L, R$ resulting in the transition $n \to n \pm 1$. These rates can be written explicitly as

$$\Gamma^+(0) = \alpha^>_L(E_-) + \alpha^<_R(eV - E_-) \tag{8}$$

$$\Gamma^-(2) = \alpha^>_L(E_+) + \alpha^>_R(eV + E_+) \tag{9}$$

$$\Gamma^+(1) = \alpha^<_L(E_+) + \alpha^<_R(eV + E_+) \tag{10}$$

$$\Gamma^-(1) = \alpha^<_L(E_-) + \alpha^>_R(eV - E_-)\ , \tag{11}$$

where $E_\pm = E_C\left[1 \pm 2(1 - Q_t/e)\right]$. It is useful to notice that at sufficiently low temperatures ($k_BT \ll \Delta$), $\Gamma^+(0)$ and $\Gamma^+(1)$ are exponentially small ($\sim e^{-\Delta/k_BT}$) while $\Gamma^-(2) \simeq \Gamma^-(1)$. From (5), it is clear that quasiparticle tunneling causes decoherence in the system. Indeed, at $Q_t = e$, the coherent oscillation between the two degenerate charge states $|0\rangle$ and $|2\rangle$ decays with the time scale $1/\Gamma \equiv 1/\Gamma^-(2) \simeq 1/\Gamma^-(1)$,

$$\sigma_0(t) \simeq \frac{1}{3} + \frac{e^{-3\Gamma t/2}}{6} + \frac{e^{-\Gamma t/2}}{2}\left(\cos\omega t + \frac{\hbar\Gamma}{E_J}\sin\omega t\right) \tag{12}$$

$$\sigma_2(t) \simeq \frac{1}{3} + \frac{e^{-3\Gamma t/2}}{6} - \frac{e^{-\Gamma t/2}}{2}\cos\omega t \tag{13}$$

$$i\chi(t) \simeq \frac{\hbar\Gamma}{3E_J} - \frac{\hbar\Gamma e^{-3\Gamma t/2}}{12E_J} + \frac{e^{-\Gamma t/2}}{4}\left(2\sin\omega t - \frac{\hbar\Gamma}{E_J}\cos\omega t\right)\ , \tag{14}$$

where the oscillation frequency is also slightly modified as $\omega \simeq (E_J/\hbar)\left[1 + (\hbar\Gamma/E_J)^2\right]$. Here we assumed the initial conditions $\sigma_0(0) = 1$ and $\sigma_2(0) = \chi(0) = 0$. In the *stationary limit* ($t \to \infty$), the charge states $|0\rangle$, $|1\rangle$, and $|2\rangle$ are equally populated. This corresponds to the Josephson-quasiparticle cycle [10, 11, 12], i.e., resonant tunneling of Cooper pairs followed by sequential tunneling of quasiparticles to the probe gate.

We now look more closely at the experiment by Nakamura *et al.* [7]. A pulse of finite length Δt was applied to the gate to change the total gate-induced charge from $Q_t = Q_0$ to $Q_t = Q_0 + Q_p$ and back. For example, if $Q_0/e < 1$ (far from the resonance point) and $(Q_0 + Q_p)/e = 1$ (at the resonance point), the pulse causes the system (which is initially in state $|0\rangle$) to oscillate between $|0\rangle$ and $|2\rangle$. Depending on the pulse length Δt, the system may or may not be in $|2\rangle$ with an increased probability at the end of the pulse. The decay of $|2\rangle$ to $|0\rangle$ through quasiparticle tunneling leads to an excess current. This was illustrated in [7] by solving the time-dependent Schrödinger equation for an isolated two-level system. Here we solve the master equation (5), which includes the decoherence due to the measuring device, and calculate the time-averaged current $I = \langle I(t)\rangle = \Gamma^-(2)\langle\sigma_2(t)\rangle + \Gamma^-(1)\langle\sigma_1(t)\rangle$. To simulate the experimental situation, we generated an array of pulses with repetition time T_r (after a few

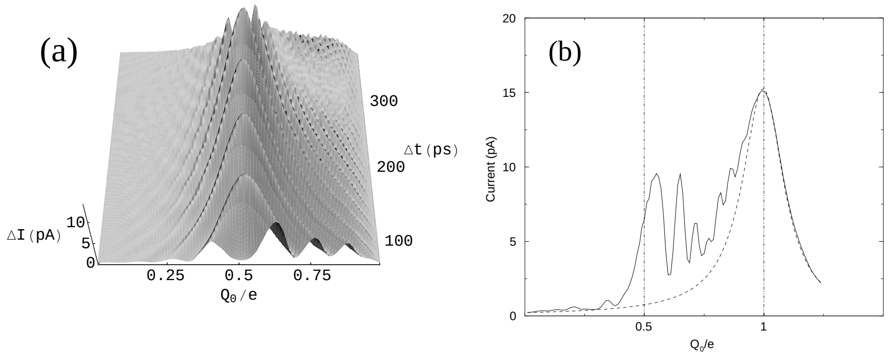

Fig. 2. – (a) Plot of pulse-induced current versus Q_0/e and Δt. (b) Current through the probe junction (R) versus Q_0/e with (solid) and without (dashed line) pulse of length $\Delta t = 2T_{coh} \simeq 160$ps. In both plots, we used the resistance ratio $R_L/R_R = 1/1800$ which corresponds to quasiparticle tunneling rates $\Gamma^-(2) \sim (6\text{ns})^{-1}$ and $\Gamma^-(1) \sim (8\text{ns})^{-1}$, i.e., like in [7].

times of repetition, the time series of $\rho(t)$ repeats the same pattern). We took the parameters used in the experiment, i.e., $E_J = 51.8\mu$eV, $E_C = 117\mu$eV, and $\Delta = 230\mu$eV. The voltage was chosen to be $eV \simeq 2\Delta + 1.65E_C$, and the repetition time $T_r = 16$ns. Figure 2 shows the result of our calculation, viz., the pulse-induced excess current versus offset gate charge Q_0/e and/or pulse length Δt.

Apart from decoherence by quasiparticle tunneling, the real experimental situation includes several complications: (i) the repetition time T_r is finite and comparable to $1/\Gamma^-(2)$ and $1/\Gamma^-(1)$; (ii) 'jittering' of the pulse; and (iii) finite rising/falling time of the pulse. The effect of these complications is summarized in Fig. 3 and they are discussed below.

Ideally, the time between two pulses should be long enough (compared to $1/\Gamma^-(2)$ or $1/\Gamma^-(1)$) such that the system is guaranteed to relax to the desired initial state. Experimentally, T_r is limited by the detector sensitivity, since the maximum value of the pulse-induced current is given by $\Delta I_{\max} = 2e/T_r$. The value of T_r in [7] was comparable to $1/\Gamma^-(2)$ or $1/\Gamma^-(1)$ and the waiting time, $T_w \equiv T_r - \Delta t$, was not long enough. First of all, this reduces the pulse-induced current collected in the probe gate with respect to the ideal maximum current $\Delta I_{\max}$. And secondly, the finite T_r also results in a wiggly behavior of $\Delta I(\Delta t)$: The charge oscillations with small amplitude and large frequency ($\sim \sqrt{E_C^2 + E_J^2}/\hbar$) have not decayed yet when the next pulse is turned on, see the time dependence of $\sigma_2(t)$ in Fig. 3 (a) (dashed line). A small change in Δt with T_r fixed leads to a change in T_w, which may be comparable to $\hbar/\sqrt{E_C^2 + E_J^2}$ and hence results in a rapid change in the pulse-induced current, see Fig. 3 (b).

The jittering of the pulse leads to fluctuations in T_r (or T_w). This smears out the wiggly behavior in ΔI discussed above. The pulse jittering in [7] was of the order of several percent of the coherence time $T_{\text{coh}} = h/E_J$ [13]. Numerically, the jittering can be taken into account by averaging $\Delta I(\Delta t)$ over slightly different values of Δt. In Fig. 2 and Fig. 3 (b) (fat solid line), we took the average over the interval $(T_r - 7\text{ps}, T_r + 7\text{ps})$.

Experimentally, the oscillation amplitude of the pulse-induced current measured in the probe gate was significantly smaller than the ideal maximum value $\Delta I_{\max}$. This may be accounted for by taking into account the finite repetition time and finite rising/falling time of the pulse. The effect of a finite repetition time was already discussed above. To understand

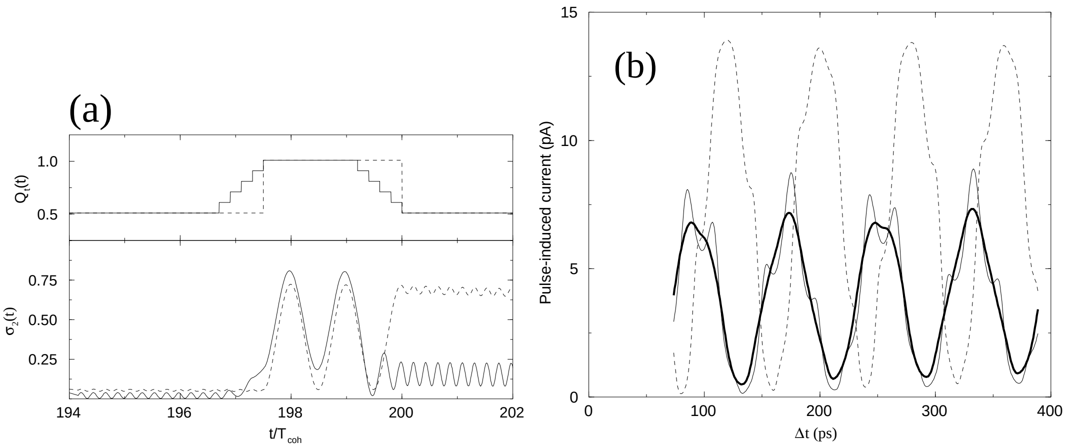

Fig. 3. – (a) Pulse shapes used to create the current in (b) and corresponding time dependence of $\sigma_2(t)$. Each step of the step-wise pulse is $0.2T_{coh}$ long. The other parameters used are $T_r = 200T_{coh} \simeq 16$ns, $Q_0/e = 0.51$, $R_L/R_R = 1/1800$ (corresponding to the values of $\Gamma^-(2) \sim (6\text{ns})^{-1}$ and $\Gamma^-(1) \sim (8\text{ns})^{-1}$ in [7]). (b) Pulse-induced current versus pulse time Δt with finite (thin solid line) and zero (dashed line) rising/falling time of the pulse. The fat solid line shows the effect of additional averaging by jittering of the pulse; its amplitude reproduces the experimental value.

the effect of the finite ramping time of the pulse, we increased/decreased the gate voltage in a step-wise way, see Fig. 3 (a). As shown in Fig. 3 (b), the oscillation amplitude for this pulse shape (thin solid line) is suppressed compared with that for an ideal pulse (dashed line). The fat solid line shows the effect of additional averaging by jittering of the pulse; its amplitude reproduces the experimental value. The shift of the oscillation in Δt is due to the change in the *effective* pulse length.

In conclusion, we have presented an analysis of the experiment by Nakamura *et al.* [7] by solving the appropriate master equation. In particular, we have considered the relevant processes involved in the detection of the charge state of the box and have used a realistic description of the gate pulse.

We would like to acknowledge stimulating discussions with Y. Nakamura, G. Falci, and E. Paladino.

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