Spectral Features of the Proximity Effect

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Abstract

We calculate the local density of states (LDOS) of a superconductor-normal metal sandwich at arbitrary impurity concentration. The presence of the superconductor induces a gap in the normal metal spectrum that is proportional to the inverse of the elastic mean free path l for rather clean systems. For a mean free path much shorter than the thickness of the normal metal, we find a gap size proportional to l that approaches the behavior predicted by the Usadel equation (diffusive limit).

Keywords: proximity effect; local density of states;

A superconductor that is in good metallic contact to a normal metal induces a finite pair amplitude on the normal side. As a result, the normal metal acquires superconducting properties like infinite conductance and the Meissner effect (see [1] and references therein). The presence of the superconductor also has consequences on the spectrum: the constant density of states around the Fermi energy of a bulk normal metal is strongly modified and shows a pseudogap for a clean normal metal. That is, it vanishes at the Fermi energy and rises linearly close to it [2]. Another well-known result on the spectrum has been obtained in the dirty (diffusive) limit [3,4]: in this case, there is a gap in the spectrum (called *minigap* to avoid confusion with the gap of the superconductor) that has the same order of magnitude as the Thouless energy $E_{Th} = \hbar D/d^2$, here D is the diffusion constant of the normal metal and d its thickness.

In related work, the spectrum of a ballistic normal cavity connected to a superconductor has been studied. A classically integrable cavity shows a LDOS that is linear in energy, whereas a chaotic cavity exhibits a gap in the spectrum [5].

The goal of the present work is to investigate a (moderately) disordered normal metal and to study in detail how the linear rise of the LDOS for the clean system transforms into the minigap in the diffusive system. To this end, we have solved the real-time Eilenberger equation for the quasi-classical 2×2 matrix Green's function $\hat{g}$ numerically for the planar geometry shown in the inset of Fig. 2 (see [1] for additional details of this method). Impurity scattering is taken into account by including an impurity self-energy of the form $\langle \hat{g} \rangle / 2\tau$ (Born approximation) determined in a self-consistent way. The SN-interface is assumed to be

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ideal, and the outer boundary of the normal metal is specularly reflecting. The normal metal is characterized by a vanishing pairing interaction. We have neglected self-consistency of the pair potential $\Delta(x)$ in the superconductor. The slight suppression of $\Delta(x)$ due to the proximity effect does not affect the low-energy spectrum of the normal metal since we assume $d \gg \xi_0 = \hbar v_F/\Delta$.

As a result of a detailed numerical study, we find that a gap forms at arbitrarily small impurity concentrations. This is shown in the upper panel of Fig. 1: even for values of the elastic mean free path l that are 50 times larger than the normal-layer thickness, the formation of the low-energy gap is clearly visible. The gap increases with $1/l$, saturates for $l \sim d$ and then decreases again as expected from the dirty-limit theory since $D \sim l$. The gap does not depend on the location in the normal metal as can be seen in the lower panel of Fig. 1, i.e., it is a global feature. The shape of the LDOS, i.e., its dependence on energy, however, varies on traversing the normal layer.

Figure 2 shows the dependence of the minigap on d/l. The behavior can be understood in a qualitative way as follows: the linear dependence of the LDOS on energy in the clean case is caused by the existence of Andreev bound states with energies $E_n(\theta) = \pi\hbar v_F(n + 1/2)\cos\theta/2d$ with $\theta \sim \pi/2$. Here, θ denotes the angle of a semiclassical trajectory with the normal vector of the interface. These trajectories (V-shaped structures bounded by two Andreev reflections and one specular reflection at the outer interface) have a length $\sim d/\cos\theta$. In almost clean systems, this length is cut off by the mean free path, hence there is a lower energy scale of $E_{Gap} \sim \hbar v_F/l \sim T_A d/l$ where $T_A = \hbar v_F/2\pi d$ is the Andreev temperature. The appropriate energy scale for the diffusive limit can be obtained as $\hbar/t$ where t is defined by $d \sim \sqrt{Dt}$. As a result, we obtain $E_{Gap} \sim \hbar D/d^2 \sim T_A l/d$.

In conclusion, we have solved the real-time Eilenberger equation numerically and have determined the local density of states of a proximity sandwich. We find a hard gap that opens for an arbitrarily small concentration of impurities and is maximal if the elastic mean free path is of the order of the normal-layer thickness.

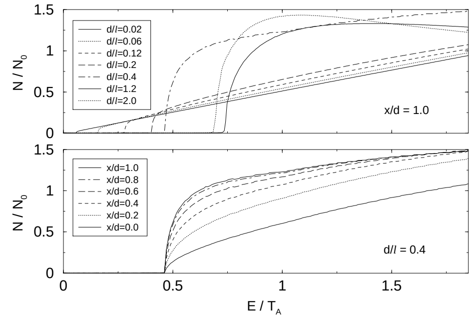

Fig. 1. Local density of states of the normal side of a proximity sandwich. d is the normal-layer thickness and l the elastic mean free path (both in N and in S); $T_A = \hbar v_F/2\pi d$; $d = 1000\xi_0$. Upper panel: Adding impurities leads to the formation of a minigap. Lower panel: The minigap is constant throughout the normal metal, but the energy dependence of the LDOS changes with location.

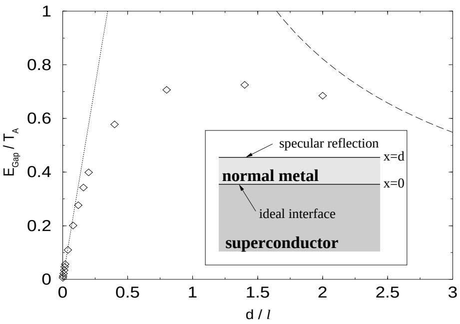

Fig. 2. Size of the minigap. Diamonds: numerical results. Dotted line: estimate in the almost clean case as given in the text. Dashed line: numerical fit to the dirty-limit results obtained in [4]. Inset: geometry of the SN-junction.

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