

Influence of dephasing on shot noise in an electronic Mach-Zehnder interferometer

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We present a general analysis of shot noise under the influence of dephasing in an electronic Mach-Zehnder interferometer, of the type recently investigated experimentally [Yang Ji *et al.*, Nature 422, 415 (2003)]. Using a model in which dephasing is caused by fluctuations of a classical field, we show that the usual partition noise expression $\mathcal{T}(1-\mathcal{T})$ is modified into a form that differs in general from both the phase-averaged product and the product of phase-averaged probabilities. In particular, we analyze the dependence on the power spectrum of the environmental fluctuations, which has not been possible in simpler phenomenological approaches like the dephasing terminal, to which we compare our results. We discuss the possibility of using shot noise as a tool to distinguish thermal smearing from genuine dephasing.

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A large part of mesoscopic physics is concerned with quantum interference effects in micrometer-size electronic circuits. Therefore, it is important to understand how interference is suppressed by the action of a fluctuating environment (such as phonons or other electrons), a phenomenon known as dephasing (or decoherence). In recent years, many experimental studies have been performed to learn more about the mechanisms of dephasing and its dependence on parameters such as temperature [1–7].

A delicate issue [4] in the analysis of interference effects other than weak localization is the fact that the “visibility” of the interference pattern can also be diminished by phase averaging: This takes place when electrons with a spread of wavelengths (determined by voltage or temperature) contribute to the current, or when some experimental parameter (such as path length) fluctuates slowly from run to run. Recently, a remarkable interference experiment has been performed using a Mach-Zehnder setup fabricated from the edge channels of a two-dimensional electron gas in the integer quantum hall effect regime [8]. Besides measuring the current as a function of the phase difference between the paths, the authors also measured the shot noise to distinguish between phase averaging and “real” dephasing. The basic idea is that both phenomena suppress the interference term in the current, but they may affect differently the partition noise which is nonlinear in the transmission probability [9]. The idea of using shot noise to learn more about dephasing is promising, connecting two fundamental issues in mesoscopic physics.

Most theoretical works on dephasing in mesoscopic interference setups are concerned with its influence on the average current only (see Refs. 10–15 and references therein), although there have been studies of shot noise in this context [16]. In this paper, we present the first analysis of shot noise for an electronic one-channel Mach-Zehnder interferometer under the influence of dephasing. We will consider dephasing produced by fluctuations of

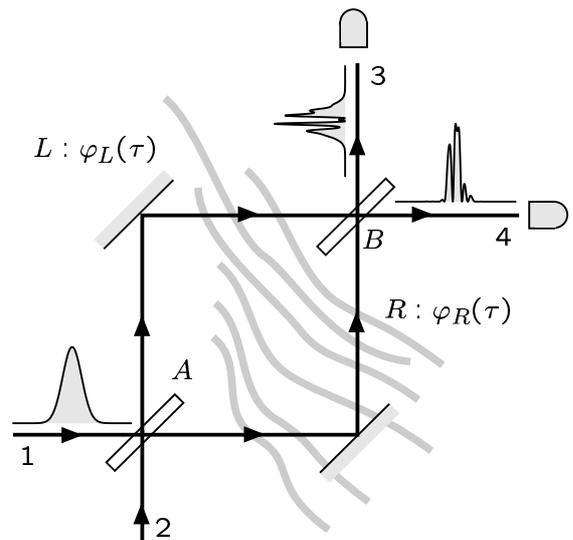


Figure 1: The Mach-Zehnder interferometer setup analyzed in the text. In the case shown here, the fluctuations of the environment are fast compared with the temporal extent of the wave packet (determined by temperature or voltage, see text). The probability density of the incoming wave packet and its two outgoing parts is shown.

a classical potential, which describe either true nonequilibrium radiation impinging on the system or the thermal part of the environmental noise. This approach has been employed quite often in the past [12, 17], is exact in the first case, and should be a reliable approximation for $T \gg eV$ in the second case. In particular, we are interested in the influence of the power spectrum of the environmental fluctuations on the shot-noise result, a question that goes beyond the phenomenological dephasing terminal model [16, 18–21].

Model and general results. - We consider non-interacting, spin-polarized electrons. By solving the Heisenberg equation of motion for the electron field Ψ

moving at constant velocity v_F (linearized dispersion relation) and under the action of a fluctuating potential $V(x, t)$ (without backscattering), we obtain

$$\hat{\Psi}(x, \tau) = \int \frac{dk}{\sqrt{2\pi}} e^{-i\epsilon_k \tau} \sum_{\alpha=1}^3 t_{\alpha}(k, \tau) \hat{a}_{\alpha}(k) e^{s_{\alpha} i k_F x} \quad (1)$$

for the electron operator at the output port 3. We have $t_3 = 1$, $s_{1,2} = 1$, $s_3 = -1$, the reservoir operators obey $\langle \hat{a}_{\alpha}^{\dagger}(k) \hat{a}_{\beta}(k') \rangle = \delta_{\alpha\beta} \delta(k - k') f_{\alpha}(k)$, and the integration is over $k > 0$ only. The amplitudes t_1, t_2 for an electron to go from terminal 1 or 2 to the output terminal 3 are time-dependent:

$$t_1(k, \tau) = t_A t_B e^{i\varphi_R(\tau)} + r_A r_B e^{i\varphi_L(\tau)} e^{i(\phi + k\delta x)} \quad (2)$$

$$t_2(k, \tau) = t_A r_B e^{i\varphi_L(\tau)} e^{i(\phi + k\delta x)} + r_A t_B e^{i\varphi_R(\tau)} \quad (3)$$

Here $t_{A/B}$ and $r_{A/B}$ are energy-independent transmission and reflection amplitudes at the two beamsplitters ($t_j^* r_j = -t_j r_j^*$), δx is a possible path-length difference, and ϕ the Aharonov-Bohm phase due to the flux through the interferometer. The electron accumulates fluctuating phases while moving along the left or right arm: $\varphi_{L,R}(\tau) = -\int_{-\tau_{L,R}}^0 dt' V(x_{L,R}(t'), \tau + t')$, where τ is the time when the electron leaves the second beamsplitter after traveling for a time $\tau_{L,R}$ along the path described by $x_{L,R}(t)$. Note that in our model the total traversal times $\tau_{L,R}$ enter only at this point, and we assumed the interaction to be confined to the interferometer region.

The output current following from (1) has to be averaged over the fluctuating phases, i.e. it depends on

phase-averaged transmission probabilities $T_1 = |t_1|^2$ and $T_2 = 1 - T_1$:

$$\langle T_1 \rangle_{\varphi} = T_A T_B + R_A R_B + 2z (r_A r_B)^* t_A t_B \cos(\phi + k\delta x), \quad (4)$$

The interference term is suppressed by $z \equiv \langle e^{i\delta\varphi} \rangle_{\varphi}$ (with $\delta\varphi = \varphi_L - \varphi_R$), which is $\exp(-\langle \delta\varphi^2 \rangle / 2)$ for Gaussian $\delta\varphi$. This decreases the visibility of the interference pattern observed in $I(\phi)$. However, such a suppression can also be brought about by the k -integration, if $\delta x \neq 0$ (thermal smearing).

Our main goal is to calculate the shot noise power S at zero frequency. It can be split into two parts:

$$S = \int d\tau \left\langle \left\langle \hat{I}(\tau) \hat{I}(0) \right\rangle \right\rangle_{\varphi} - \left\langle \left\langle \hat{I}(0) \right\rangle \right\rangle_{\varphi}^2 = \int d\tau \left\langle \left\langle \hat{I}(\tau) \right\rangle \left\langle \hat{I}(0) \right\rangle \right\rangle_{\varphi} - \left\langle \left\langle \hat{I}(0) \right\rangle \right\rangle_{\varphi}^2 + \int d\tau \left\langle \left\langle \hat{I}(\tau) \hat{I}(0) \right\rangle - \left\langle \hat{I}(\tau) \right\rangle \left\langle \hat{I}(0) \right\rangle \right\rangle_{\varphi} \quad (5)$$

The first integral on the r.h.s. describes shot noise due to the temporal fluctuations of the conductance, i.e. fluctuations of a classical current $I(\tau) = \langle \hat{I}(\tau) \rangle$ depending on time-dependent transmission probabilities. We denote its noise power as S_{cl} . It rises quadratically with the total current, as is known from $1/f$ -noise in mesoscopic conductors [22].

The second integral is evaluated by inserting (1) and applying Wick's theorem (similar formulas appear in Ref. [23]):

$$\left\langle \left\langle \hat{I}(\tau) \hat{I}(0) \right\rangle - \left\langle \hat{I}(\tau) \right\rangle \left\langle \hat{I}(0) \right\rangle \right\rangle_{\varphi} = \left(\frac{e v_F}{2\pi} \right)^2 \int dk dk' \sum_{\alpha, \beta=1,2,3} f_{\alpha}(k) (1 - f_{\beta}(k')) K_{\alpha\beta}(\tau) e^{i(\epsilon_{k'} - \epsilon_k)\tau}. \quad (6)$$

Here $K_{\alpha\beta}$ is a correlator of four transmission amplitudes: We have $K_{33} = 1$, $K_{3\alpha} = K_{\alpha 3} = 0$, and

$$K_{\alpha\beta}(\tau) \equiv \langle t_{\alpha}^*(k, \tau) t_{\beta}(k', \tau) t_{\alpha}(k, 0) t_{\beta}^*(k', 0) \rangle_{\varphi}, \quad (7)$$

for $\alpha, \beta = 1, 2$.

In order to obtain two limiting forms of this expression, we note that the τ -range of the oscillating exponential factor under the integral in (6) is determined by the Fermi functions, i.e. by voltage and temperature. This has to be compared with the correlation time τ_c of the environment (the typical decay time of the phase correlator $\langle \delta\varphi(\tau) \delta\varphi(0) \rangle$). For $eV\tau_c \ll 1$ and $T\tau_c \ll 1$ ("fast environment"), the major contribution of the integration

comes from $|\tau| \gg \tau_c$, where $K_{\alpha\beta}$ factorizes into

$$K_{\alpha\beta}(\tau) \approx K_{\alpha\beta}(\infty) \equiv \left| \langle t_{\alpha}^*(k, 0) t_{\beta}(k', 0) \rangle_{\varphi} \right|^2. \quad (8)$$

This yields the noise power

$$\frac{S_{\text{fast}}}{e^2 v_F / 2\pi} = \int dk \sum_{\alpha, \beta=1,2} f_{\alpha}(1 - f_{\beta}) \left| \langle t_{\alpha}^* t_{\beta} \rangle_{\varphi} \right|^2 + f_3(1 - f_3), \quad (9)$$

where we have set $f_{\alpha, \beta} = f_{\alpha, \beta}(k)$ and $t_{\alpha, \beta} = t_{\alpha, \beta}(k, 0)$. We conclude that the shot noise for a "fast" environment

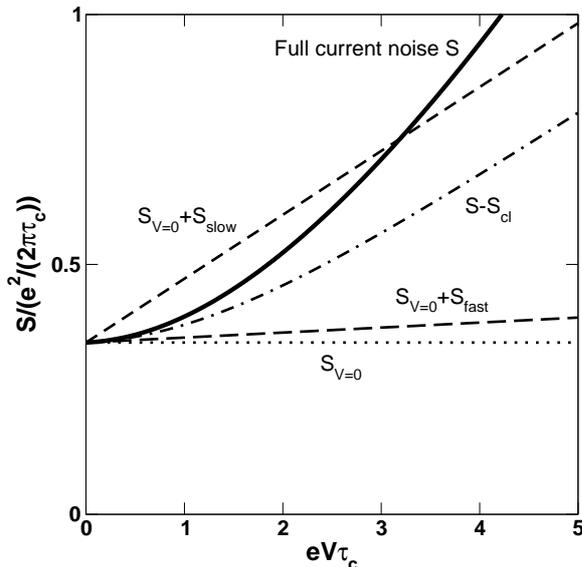


Figure 2: Typical behaviour of the full current noise S as a function of $eV\tau_c$. At higher voltages, the dependence on V is quadratic, due to S_{cl} . When S_{cl} is subtracted, the slope at large $eV\tau_c$ is determined by S_{slow} (i.e. $\langle T_1(1-T_1) \rangle_\varphi$), while that at low voltages is always determined by S_{fast} (i.e. $|\langle t_1^* t_2 \rangle_\varphi|^2$). Parameters: $T = 0$, $\delta x = 0$, $\phi = 0$, $T_A = 1/2$, $z = 1/e$, $T_B = 0.4$.

is *not* given by an expression of the form $\langle T \rangle_\varphi (1 - \langle T \rangle_\varphi)$, which would be obtained from a simple classical model (see the discussion at the end of this article). Indeed, we have

$$\left| \langle t_1^* t_2 \rangle_\varphi \right|^2 - \langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi) = (z^2 - 1) R_B T_B. \quad (10)$$

The remainder of the noise power from Eq. (6) (with $K_{\alpha\beta}(\tau) - K_{\alpha\beta}(\infty)$ inserted in Eq. (6)) will be denoted S_{fluct} . It yields a contribution to the Nyquist noise $S_{V=0}$, but apart from that it becomes important only at larger V , T . With this definition, the full noise power can always be written as

$$S = S_{fast} + S_{fluct} + S_{cl}. \quad (11)$$

In the other limiting case, when the τ -integration is dominated by $|\tau| \ll \tau_c$ (“slow environment”), we can use $K_{\alpha\beta}(\tau) \approx K_{\alpha\beta}(0)$, which yields

$$\frac{S_{slow}}{e^2 v_F / 2\pi} = \int dk \langle (f_1 T_1 + f_2 T_2)(1 - (f_1 T_1 + f_2 T_2)) \rangle_\varphi + f_3(1 - f_3), \quad (12)$$

i.e. the phase-average of the usual shot noise expression (at $T = 0$ the integrand is $\langle T_1(1 - T_1) \rangle_\varphi$).

Discussion. - We are able to evaluate the phase-averages if the potential $V(x, t)$ (and therefore $\delta\varphi$) is assumed to be a Gaussian random field of zero mean. In the following, we present explicit expressions for the case $T = 0$, $\delta x eV / v_F \ll 1$, where the visibility is decreased purely by dephasing. We can express the results by the following Fourier transforms that depend on the power spectrum of the fluctuations and are nonperturbative in the fluctuating field ($\lambda = \pm$):

$$\hat{g}_\lambda(\omega) \equiv \int d\tau e^{i\omega\tau} [e^{\lambda \langle \delta\varphi(\tau)\delta\varphi(0) \rangle} - 1], \quad (13)$$

$$I_\lambda(V) \equiv \int_0^{eV} d\omega \left(1 - \frac{\omega}{eV}\right) \hat{g}_\lambda(\omega) \quad (14)$$

The shot noise becomes ($\tilde{\phi} = \phi + k_F \delta x$):

$$\begin{aligned} \frac{S - S_{V=0}}{e^3 V / 2\pi} &= \frac{eV}{\pi} z^2 R_A R_B T_A T_B (\cos(2\tilde{\phi}) \hat{g}_-(0) + \hat{g}_+(0)) \\ &+ \left| \langle t_1^* t_2 \rangle_\varphi \right|^2 \\ &+ \frac{1}{\pi} z^2 R_B T_B \left\{ -2 \cos(2\tilde{\phi}) R_A T_A I_-(V) \right. \\ &\left. + (R_A^2 + T_A^2) I_+(V) \right\} \end{aligned} \quad (15)$$

The first line corresponds to S_{cl} , the second to S_{fast} , and the rest to $S_{fluct} - S_{V=0}$. At $V \rightarrow 0$, the integrals $I_\pm(V)$ vanish and S_{fast} dominates. At large $eV\tau_c \gg 1$ we can use the sum-rule $I_\lambda(V) \rightarrow \pi [z^{-2\lambda} - 1]$ and find the last three lines to combine to $\langle T_1(1 - T_1) \rangle_\varphi$, i.e. S_{slow} . The Nyquist noise is ϕ -independent:

$$S_{V=0} = \frac{e^2}{2\pi^2} z^2 R_B T_B \int_0^\infty d\omega \omega \hat{g}_+(\omega). \quad (16)$$

The results are illustrated in Figs. 2 and 3, where the evolution of S with increasing voltage V shows the cross-over between a “fast” and a “slow” environment. Although S_{fast} can become zero, the total current noise S does not vanish, due to the Nyquist contribution. The plots have been produced by assuming 50% transparency of the first beamsplitter ($T_A = 1/2$) and using a simple Gaussian form for the phase correlator:

$$\langle \delta\varphi(\tau)\delta\varphi(0) \rangle = \langle \delta\varphi^2 \rangle e^{-(\tau/\tau_c)^2}. \quad (17)$$

An application of the general theory presented here to specific situations includes the calculation of the phase-correlator, starting from the correlator $\langle VV \rangle_{q\omega}$ describing the potential fluctuations $V(x, t)$ (cf. [12] for an example). The contribution of potential modes to $\langle \delta\varphi^2 \rangle$ is suppressed for $|q| < 1/R$ (R : typical distance between the paths) and becomes maximal for small $|v_F q - \omega|$.

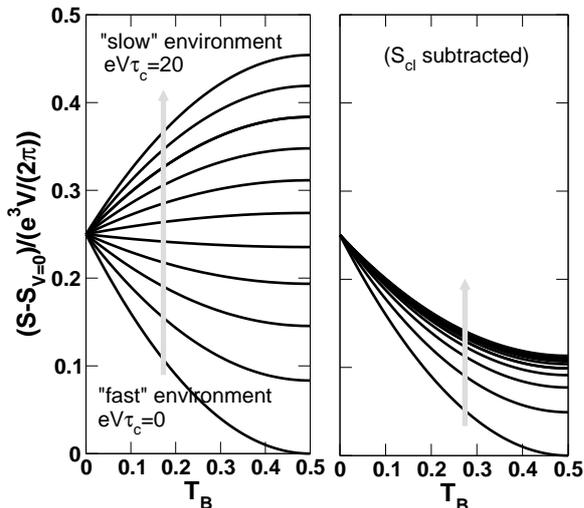


Figure 3: Normalized shot noise $(S - S_{V=0}) / (e^3 V / 2\pi)$ as a function of the transmission T_B of the second beamsplitter for small visibility ($z = 1/e$). The different curves show the succession from a “fast” environment to a “slow” one (bottom to top : $eV\tau_c = 0, 2, 4, \dots, 20$). In the right panel, the contribution from S_{cl} (first line of (15)) has been dropped, to demonstrate the convergence against the result for a slow bath, $\langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi)$ (topmost curve). Other parameters as in Fig. 2.

If shot noise is to be used as a tool to distinguish phase averaging from ‘genuine dephasing’, it is important to inquire about the form of shot noise for a situation in which both effects are present. In particular, for a large path-length difference $\delta x \gg v_F / eV$, or $\delta x \gg v_F / T$, the interference term is already suppressed completely due to wavelength averaging. Then S_{cl} vanishes, since $\langle \hat{I}(\tau) \rangle$ is independent of the fluctuating phase. In addition, we find (at $T = 0$):

$$\frac{S - S_{V=0}}{e^3 V / 2\pi} = T_A R_A (T_B - R_B)^2 + z^2 T_B R_B (T_A^2 + R_A^2) \left[1 + \frac{I_+(V)}{\pi} \right]. \quad (18)$$

For a “fast” environment, we have $I_+(V) \rightarrow 0$, such that Eq. (18) becomes $[(T_B - R_B)^2 + 2z^2 R_B T_B] / 4$, which turns into $(T_B - R_B)^2 / 4$ for $z \rightarrow 0$. This could be distinguished from the k -averaging result, but it corresponds to the case of large energy transfers from and to the environment (as opposed to “pure dephasing”). On the other hand, in the limit of large voltages (“slow environment”, $eV\tau_c \gg 1$), we have $I_+(V) \rightarrow \pi [z^{-2} - 1]$, and Eq. (18) turns into $(T_B^2 + R_B^2) / 4$, which is equal to the result obtained for pure k -averaging (without any additional dephasing).

We have pointed out already even S_{fast} does not lead to the simple result $\langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi)$. However, this form does indeed apply if we consider injecting only a “narrow

beam” of electrons into terminal 1 (i.e. $f_\alpha(k) = 0$ except for $f_1(k) = 1$ within $[k_F, k_F + eV / v_F]$), which is not equivalent to the previous situation regarding shot noise (cf. [24] in this respect). We get for $eV\tau_c \ll 1$:

$$S - S_{cl} = \frac{e^3 V}{2\pi} \langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi), \quad (19)$$

while the case $eV\tau_c \gg 1$ is described by S_{slow} .

Comparison with other models. - Finally, we compare our results in the fully incoherent limit ($z = 0$) with two other models, namely the phenomenological dephasing terminal [16, 20, 21], and a simple model of a stream of regularly injected electrons [25] that reach the output port with a probability calculated according to classical rules. We focus on $T = 0$ and the case $T_A = 1/2$. At small path-length difference $eV\delta x / v_F \ll 1$ (no k -averaging), we obtain $\langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi) = 1/4$ both for the classical model and the narrow beam of electrons, $(T_B - R_B)^2 / 4$ for our shot-noise expression in the “fast” case, and $(T_B^2 + R_B^2) / 4$ both for the “slow” case and from the dephasing terminal [26]. In the opposite limit of large δx only the result for the classical model changes, coinciding with the “slow” case $(T_B^2 + R_B^2) / 4$, which is also obtained without any dephasing. Therefore, in this regime a shot noise measurement most likely will not be able to reveal the additional presence of dephasing.

In conclusion, we have analyzed the effects of dephasing on shot noise in an experimentally relevant model of an electronic Mach-Zehnder interferometer. We have generalized the scattering theory of shot noise to include dephasing induced by fluctuations of a classical potential. This has enabled us to analyze the dependence of shot noise on the power spectrum of the fluctuations, going beyond simpler phenomenological approaches, to which we have compared our results. We have identified a crossover between two regimes, those of a “fast” and a “slow” environment. Finally, we have pointed out that a shot-noise measurement cannot reveal the presence of dephasing on top of thermal averaging (related to a finite path-length difference), for environmental fluctuations slower than the inverse voltage or temperature. Our theory may be applied to other single-channel interferometer geometries as well, even in the presence of backscattering at the junctions.

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[1] P. Mohanty, E. M. Q. Jariwala, and R. A. Webb, Phys. Rev. Lett. **78**, 3366 (1997).

- [2] E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and H. Shtrikman, *Nature* **391**, 871 (1998).
- [3] D. P. Pivin, A. Andresen, J. P. Bird, and D. K. Ferry, *Phys. Rev. Lett.* **82**, 4687 (1999).
- [4] A. E. Hansen, A. Kristensen, S. Pedersen, C. B. Sørensen, and P. E. Lindelof, *Phys. Rev. B* **64**, 045327 (2001).
- [5] D. Natelson, R. L. Willett, K. W. West, and L. N. Pfeiffer, *Phys. Rev. Lett.* **86**, 1821 (2001).
- [6] K. Kobayashi, H. Aikawa, S. Katsumoto, and Y. Iye, *J. Phys. Soc. Jpn.* **71**, L2094 (2002).
- [7] F. Pierre and N. O. Birge, *Phys. Rev. Lett.* **89**, 206804 (2002).
- [8] Y. Ji, Y. Chung, D. Sprinzak, M. Heiblum, D. Mahalu, and H. Shtrikman, *Nature* **422**, 415 (2003).
- [9] V. A. Khlus, *Zh. Eksp. Teor. Fiz.* **93**, 2179 (1987); G. B. Lesovik, *JETP Lett.* **49**, 592 (1989).
- [10] I. L. Aleiner, N. S. Wingreen, and Y. Meir, *Phys. Rev. Lett.* **79**, 3740 (1997).
- [11] Y. Levinson, *Europhys. Lett.* **39**, 299 (1997).
- [12] G. Seelig and M. Büttiker, *Phys. Rev. B* **64**, 245313 (2001).
- [13] F. Marquardt and C. Bruder, *Phys. Rev. B* **65**, 125315 (2002).
- [14] F. Marquardt and C. Bruder, *cond-mat/0303397* (2003).
- [15] G. Seelig, S. Pilgram, A. N. Jordan, and M. Büttiker, *cond-mat/0304022* (2003).
- [16] J. H. Davies, J. C. Egues, and J. W. Wilkins, *Phys. Rev. B* **52**, 11259 (1995); C. Texier and M. Büttiker, *Phys. Rev. B* **62**, 7454 (2000).
- [17] B. L. Altshuler, A. G. Aronov, and D. E. Khmel'nitsky, *J. Phys. C Solid State* **15**, 7367 (1982); S. Chakravarty and A. Schmid, *Phys. Rep.* **140**, 195 (1986); A. Stern, Y. Aharonov, and Y. Imry, *Phys. Rev. A* **41**, 3436 (1990).
- [18] M. Büttiker, *Phys. Rev. B* **33**, 3020 (1986).
- [19] M. J. M. de Jong and C. W. J. Beenakker, in *Mesoscopic Electron Transport*, ed. by L. P. Kouwenhoven, L. L. Sohn, and G. Schön, NATO ASI Series Vol. 345 (Kluwer Academic, Dordrecht, 1997).
- [20] M.J.M. de Jong and C. W. J. Beenakker, *Physica A* **230**, 219 (1996).
- [21] S. A. van Langen and M. Büttiker, *Phys. Rev. B* **56**, R1680 (1997).
- [22] Sh. Kogan, *Electronic noise and fluctuations in solids* (Cambridge Univ. Press, Cambridge 1996).
- [23] G. B. Lesovik, *JETP Lett.* **70**, 208 (1999).
- [24] U. Gavish, Y. Levinson, and Y. Imry, *Phys. Rev. Lett.* **87**, 216807 (2001).
- [25] Ya. M. Blanter and M. Büttiker, *Phys. Rep.* **336**, 1 (2000).
- [26] There are indications [27] that the ansatz used for calculating shot noise in the dephasing terminal approach underestimates shot noise in the limit of small δx . Obtaining the proper result for the classical model at large δx requires taking into account both the anticorrelations of inputs and exchange scattering effects at the second beam splitter (both of which reduce shot noise at intermediate T_B).
- [27] F. Marquardt and C. Bruder, in preparation.