Josephson effect in $S_F X S_F$ junctions

N. M. Chtchelkatchev[+1)], W. Belzig*, and C. Bruder*

$^+$*L.D. Landau Institute for Theoretical Physics RAS, 117940 Moscow, Russia*

*Departement Physik und Astronomie, Universität Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland

We investigate the Josephson effect in $S_F X S_F$ junctions, where S_F is a superconducting material with a ferromagnetic exchange field, and X a weak link. The critical current I_c increases with the (antiparallel) exchange fields if the distribution of transmission eigenvalues of the X-layer has its maximum weight at small values. This exchange field enhancement of the supercurrent does not exist if X is a diffusive normal metal. At low temperatures, there is a correspondence between the critical current in an $S_F I S_F$ junction with collinear orientations of the two exchange fields, and the AC supercurrent amplitude in an SIS tunnel junction. The difference of the exchange fields $h_1 - h_2$ in an $S_F I S_F$ junction corresponds to the potential difference $V_1 - V_2$ in an SIS junction; i.e., the singularity in I_c [in an $S_F I S_F$ junction] at $|h_1 - h_2| = \Delta_1 + \Delta_2$ is the analogue of the Riedel peak. We also discuss the AC Josephson effect in $S_F I S_F$ junctions.

PACS: 74.50.+r, 74.80.-g, 75.70.-i

The presence of a magnetic exchange field in bulk superconductors [1, 2] and in superconductor (S) – ferromagnet(F) multilayers reduces the critical temperature T_c, i.e., suppresses superconductivity (see, e.g., [3] and references therein). Similarly, an exchange field suppresses the proximity effect: superconducting correlations spread into the F layer of superconductor-ferromagnet structures on a shorter distance than into the normal layer of a superconductor-normal metal structure [4]. Hence, it is natural to expect that the supercurrent in a junction will be suppressed by an exchange field in the superconductors or by the presence of ferromagnetic layers between the superconducting banks. Surprisingly, it was shown recently that the supercurrent can be strongly enhanced in a number of situations: e.g., in an $S_F I S_F$ junction formed by two "ferromagnetic superconductors" (S_F) whose exchange fields are oriented in an antiparallel way [5, 6], and in SFIFS junctions [7, 8]. There is still no simple intuitive understanding of this exchange field supercurrent enhancement (EFSE) effect, and also, which conditions favor this effect. In the following, we investigate the Josephson effect in $S_F X S_F$ junctions for different choices of the scattering layer X, for example, when X is a diffusive normal metal or an insulator, and find the conditions favoring the EFSE-effect.

In this Letter, we show that the EFSE-effect exists in $S_F X S_F$ junctions if the distribution of transmission eigenvalues of the X-layer has its maximum weight for small values. If the transparency is increased, we
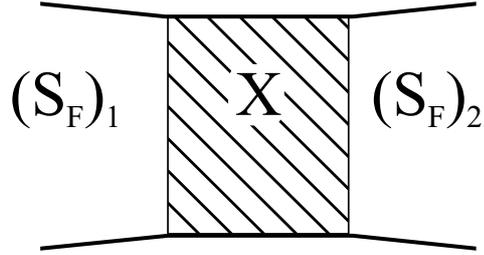
Fig. 1 Sketch of the device showing the EFSE-effect; the two ferromagnetic superconductor layers S_F are characterized by BCS order parameters $\Delta_{1(2)}$ and exchange fields $\mathbf{h}_{1(2)}$. A scattering region X (e.g., an insulator or a diffusive normal metal) separates the two S_F layers.

find that the effect becomes less pronounced; it disappears when the transparency is close to unity. If X is a diffusive normal metal, there is no exchange field enhancement of the supercurrent. At zero temperature, we find a correspondence between the critical current $I_c(V = 0, h_1 - h_2)$ of an $S_{F1} I S_{F2}$ junction with collinear exchange fields $h_{1(2)}$ and the AC supercurrent amplitude $\mathrm{Re}\, I_c(V)$ of an SIS tunnel junction. The two quantities coincide if the voltage V across the junction is equal to $h_1 - h_2$. Thus, the peak-like singularity of $I_c(V = 0, h_1 - h_2)$ at $|h_1 - h_2| = \Delta_1 + \Delta_2$ has the same nature as the Riedel peak in SIS contacts at $|eV| = \Delta_1 + \Delta_2$ [9–12]. Here, $\Delta_{1(2)}$ are the superconducting pair potentials of the two contacts.

To derive the results listed above, we relate the supercurrent through the $S_F X S_F$ junction to the scattering matrix of the region X, and then use the statistical properties of this scattering matrix. The model considered is illustrated in Fig. 1. It consists of a scattering region (hatched) between two superconducting S_F layers.

Examples for S_F-layers include superconductors with ferromagnetic impurities [1], or superconductor-ferromagnet (normal metal) multilayers, where the superconducting (and ferromagnetic) order parameter is induced by the proximity effect [5, 13]. They can be described by adding an exchange field to the BCS-model [14, 15]. Then the self-consistency equation at zero temperature shows that the superconducting order parameter $\Delta(h) = \Delta(0)$ if the exchange field $h < \Delta(0)$, and $\Delta(h) = 0$ otherwise. In this paper we assume that $|h| \leq \Delta(0)$ in the two "ferromagnetic superconductor" leads.

The supercurrent is calculated using the quasiclassical Green's function technique. We assume that the junction is short, i.e., that the traversal time τ through the region X is such that $\hbar/\tau$ exceeds the superconducting order parameters $\Delta_{1,2}$ of the S_F layers. Then, following [16, 17], we relate the supercurrent I to Keldysh Green's functions, and finally to retarded quasiclassical Green's functions $\hat{R}_{1,2}$ in the bulk of the S_F layers, and the eigenvalues $\mathcal{T}_n$ of $tt^\dagger$, where t is the transmission amplitude of the X-layer:

$$I = \frac{1}{4e} \int dE \, \text{Tr} \left[\hat{\tau}^{(3)} \hat{I}(E) \right] \tanh \left(\frac{E}{2T} \right) , \quad (1a)$$

$$\hat{I} = \frac{e^2}{\pi \hbar} \sum_n 2\mathcal{T}_n \frac{[\hat{R}_1, \hat{R}_2]}{4 + \mathcal{T}_n(\{\hat{R}_1, \hat{R}_2\} - 2)} , \quad (1b)$$

$$\hat{R}_{1,2}(E) = \frac{i}{\sqrt{(\Delta_{1,2})^2 - (E + \vec{\sigma} \cdot \mathbf{h}_{1,2})^2}} \times \quad (1c)$$

$$\times \begin{pmatrix} E + \vec{\sigma} \cdot \mathbf{h}_{1,2} & e^{i\varphi_{1,2}} \Delta_{1,2} \\ -e^{-i\varphi_{1,2}} \Delta_{1,2} & -E - \vec{\sigma} \cdot \mathbf{h}_{1,2} \end{pmatrix} .$$

Here, $\hat{\tau}^{(3)}$ is the Pauli matrix acting in Nambu space, the trace is taken over the Nambu and spin-spaces, and $\varphi_{1,2}$ is the superconducting phase corresponding to the S_F layers. Equations (1a-1c) are valid for both ballistic or dirty S_F-layers.

To derive Eqs. (1a,1b) we used the general Zaitsev boundary conditions [16, 17] for the Green's functions rather than the Kupriyanov-Lukichev dirty-limit approximation [18] which is valid for small $\mathcal{T}$ (see, e.g., [19] and references therein). Using the Zaitsev boundary condition leads to the anticommutator of the Green's functions in the denominator of Eq. (1b) which plays an important role here and cannot be neglected. Due

to this anticommutator the EFSE-effect is suppressed in $S_F I S_F$ junctions with large transparencies $\mathcal{T}$ and in $S_F X S_F$ junctions in which X is a dirty normal metal, see, e.g., Fig. 3.

If $\mathbf{h}_1 \| \mathbf{h}_2$, Eq. (1b) reduces to:

$$I(\varphi) = \sum_{\sigma = \pm 1} \int d\mathcal{T} \rho(\mathcal{T}) \frac{e}{\hbar} T \sum_\omega \frac{d}{d\varphi} \ln[g(i\omega, \varphi, \sigma, \mathcal{T})] .$$

$$(2)$$

Here,

$$g(E, \varphi, \sigma, \mathcal{T}) = (1 - \mathcal{T}) \sin(a_1) \sin(a_2) +$$

$$\frac{1}{2} \mathcal{T} (\cos(\varphi) - \cos(a_1 + a_2)) , \quad (3)$$

where $\varphi = \varphi_1 - \varphi_2$, $\rho(\mathcal{T}) = \sum_n \delta(\mathcal{T} - \mathcal{T}_n)$ is the distribution of transmission eigenvalues, $\omega = 2\pi T(k + 1/2)$, $k = 0, \pm 1, \ldots$ are Matsubara frequencies, and $a_{1,2} = \arccos[(E + \sigma h_{1,2})/\Delta_{1,2}]$ represent the phases picked up at an Andreev reflection from the S_F layers. Equations (2) and (3) can be also derived using the scattering theory developed in Ref. [20].

In the general case $\mathbf{h}_1 \nparallel \mathbf{h}_2$, the supercurrent is given by

$$I(\varphi) = I^{(p)}(\varphi) \cos^2 \left(\frac{\theta}{2} \right) + I^{(a)}(\varphi) \sin^2 \left(\frac{\theta}{2} \right) , \quad (4)$$

where the indices p, a correspond to the parallel and antiparallel configurations of the exchange fields; θ is the angle between $\mathbf{h}_1$ and $\mathbf{h}_2$. Equation (4) can be derived from (1a)-(1c) using the following identity for an analytical function L of two variables:

$$\text{Tr} \, L[(\vec{\sigma} \cdot \mathbf{a}), (\vec{\sigma} \cdot \mathbf{b})] \equiv$$

$$\frac{1}{2} \sum_{\sigma_{1(2)} = \pm 1} \left(1 + \sigma_1 \sigma_2 \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \right) L[\sigma_1 |\mathbf{a}|, \sigma_2 |\mathbf{b}|] , \quad (5)$$

where the trace is taken over spin degrees of freedom. The last identity can be proved by a series expansion.

Using Eqs. (2-4), we can work out the effect of ferromagnetic interactions on the supercurrent in a number of structures.

We shall concentrate below on the case when the exchange fields are collinear. Suppose that X is a tunnel barrier. Then $\rho(\mathcal{T}) = N\delta(\mathcal{T} - D)$, where $D \ll 1$, N is the number of channels [$N = k_F^2 A/4\pi$, where A is the area of the junction cross-section, and k_F is the Fermi wave vector in S]. It follows from (2) that

$$I(\varphi) = \sin(\varphi) \frac{\pi}{e} (R_N)^{-1} \Delta_1 \Delta_2 \times \quad (6)$$

$$T \sum_\omega \text{Re} \frac{1}{\sqrt{(\Delta_1)^2 + (\omega + ih_1)^2} \sqrt{(\Delta_2)^2 + (\omega + ih_2)^2}} ,$$
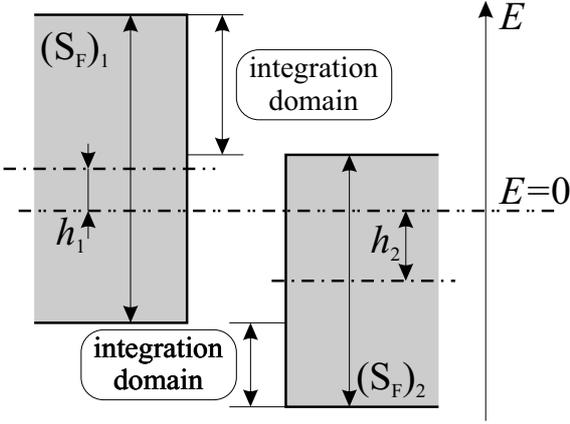
Fig. 2 The integration domain shown gives the main contribution to the supercurrent in an $(S_F)_1 I (S_F)_2$ junction according to Eq. (7). The supercurrent shows a Riedel singularity when $|h_1| \to \Delta_1$, $|h_2| \to \Delta_2$.

where $R_N = (NDe^2/\pi\hbar)^{-1}$ is the normal-state resistance of the junction. If sign$(h_1 h_2) > 0$, Eq. (6) gives $I^{(p)}$, and in the opposite case $I^{(a)}$ [see Eq. (4)]. For $\Delta_1 = \Delta_2$, $h_1 = -h_2$, Eq. (6) reproduces the corresponding results of [5].

It follows from Eq. (6) that at small temperatures, $T \ll \min\{\Delta_1, \Delta_2\}$, as long as $|h_1| < \Delta_1, |h_2| < \Delta_2$, the supercurrent does not depend on $h_1 + h_2$. It grows with $h_1 - h_2$ and diverges logarithmically when $|h_1 - h_2| \to \Delta_1 + \Delta_2$. To illustrate this, we write Eq. (6) in the real-time representation:

$$I(\varphi) = \frac{\Delta_1 \Delta_2}{4 R_N} \sin(\varphi) \sum_{\sigma = \pm 1} \int_{-\infty}^{\infty} dE \tanh\left(\frac{E}{2T}\right) \times \quad (7)$$

$$\text{Im} \frac{1}{\sqrt{((\Delta_1)^2 - (E + h_1 \sigma)^2)((\Delta_2)^2 - (E + h_2 \sigma)^2)}} .$$

The integration domain is shown in Fig. 2. Equation (7) and Fig. 2 show that the exchange fields $h_{1(2)}$ shift the Fermi energies of the two superconductors by $\sigma h_{1(2)}$. The potentials $V_{1(2)}$ applied to the superconducting banks of an SIS junction shift the Fermi energies in a similar manner. In particular, it turns out that the amplitude $\text{Re } I_c(V)$ of the AC Josephson supercurrent [which is proportional to $\sin(2eVt/\hbar)$] of an SIS junction is equal to the critical current $I_c = I(\varphi = \pi/2)$ in Eqs. (6,7) after the substitution $h_{1(2)} \to eV_{1(2)}$. At zero temperature, the critical current $I_c = I(\varphi = \pi/2)$ defined by Eq. (7) can be expressed through the elliptic

function **K** [10, 11, 21]. If we define $h \equiv h_1 - h_2$, then within the interval $|h| < |\Delta_1 - \Delta_2|$,

$$I_c R_N = \frac{2e\Delta_1 \Delta_2}{\sqrt{(\Delta_1 + \Delta_2)^2 - h^2}} \times$$
$$\mathbf{K}\left(\sqrt{\frac{(\Delta_1 - \Delta_2)^2 - h^2}{(\Delta_1 + \Delta_2)^2 - h^2}}\right) . \quad (8)$$

If $|\Delta_1 - \Delta_2| < |h| < \Delta_1 + \Delta_2$ then

$$I_c R_N = e\sqrt{\Delta_1 \Delta_2} \, \mathbf{K}\left(\sqrt{\frac{4\Delta_1 \Delta_2}{h^2 - (\Delta_1 - \Delta_2)^2}}\right) . \quad (9)$$

For $h_1 = h_2 = 0$, $\Delta_1 = \Delta_2$, Eq. (9) leads to $I_c R_N = e\Delta\pi/2$, i.e., the usual result of the critical current of an SIS Josephson junction [12].

For $|h|$ close to $\Delta_1 + \Delta_2$, the integral (7) has a singularity. The singular part of the current is:

$$I_c R_N \sim \frac{e\sqrt{\Delta_1 \Delta_2}}{2} \ln\left(\frac{\Delta_1 + \Delta_2}{||h| - (\Delta_1 + \Delta_2)|}\right) . \quad (10)$$

If the temperature is close to the critical temperature of the S_F layer, the supercurrent depends on $h_1 + h_2$ as well as $h_1 - h_2$, and there is no EFSE-effect in agreement with [5]. In this case, the correspondence of the exchange field in $S_F X S_F$ junctions and the voltage in SIS junctions is not valid any more.

The main point of the discussion above is that the supercurrent is strongly enhanced by the exchange field in the tunnelling regime, i.e., when the scattering region X is an insulator with small transparency. Below we investigate whether the enhancement effect is seen in other types of $S_F X S_F$ junctions, e.g., when the layer X is a diffusive normal metal.

If $\Delta = \Delta_1 = \Delta_2$, $h \equiv h_1 = -h_2$ (antiparallel magnetizations), Eq. (3) can be simplified:

$$g(E, \varphi, \sigma, \mathcal{T}) = \frac{2 - \mathcal{T}}{2\Delta^2}\sqrt{(\Delta^2 - E^2 - h^2)^2 - 4E^2 h^2} +$$
$$\frac{\mathcal{T}}{2}\left(\cos(\varphi) + \frac{h^2 - E^2}{\Delta^2}\right) . \quad (11)$$

The current can be evaluated using (2).

Let us first turn to the case when the distribution of transmission eigenvalues $\rho \propto \delta(\mathcal{T} - D)$. As shown above, the enhancement effect exists as long as $D \ll 1$. If the transparency D becomes larger, we find from Eq. (2) that the EFSE-effect becomes less pronounced; it disappears when the transparency is close to unity. This is illustrated in Fig. 3a, where the critical current of an $S_F X S_F$ junction with $\Delta \equiv |\Delta_1| = |\Delta_2|$ is shown as a function of the exchange field $E_{ex} \equiv h_1 = -h_2$ at
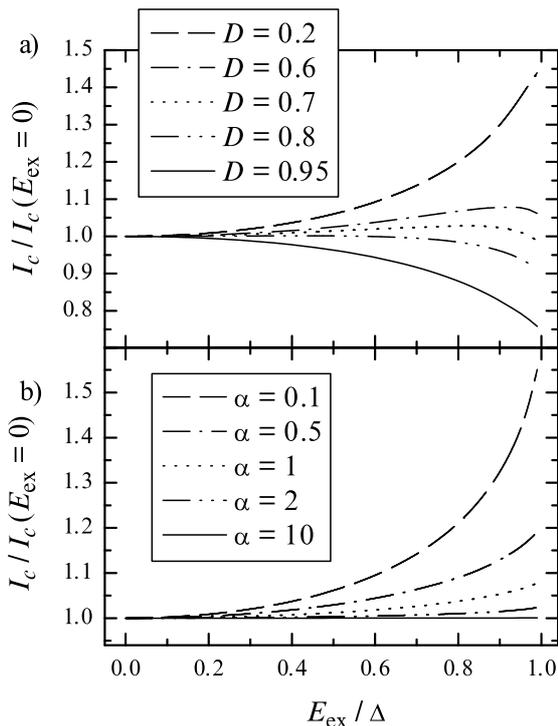
Fig. 3 Exchange field dependence of the critical current in an $S_F X S_F$ junction with $\Delta_1 = \Delta_2$ and $E_{ex} \equiv h_1 = -h_2$. (a) X is an insulator with transparency D. For $D > 0.7$, the supercurrent enhancement effect disappears. (b) X is a disordered normal metal of conductance G_N with a tunnel junction of conductance G_T; $\alpha = G_T/G_N$. The supercurrent enhancement effect disappears for $\alpha \gg 1$.

different transparencies D. The relation of the transparency and the normal state resistance is given by $D = R_{Sh}/R_N$, where the Sharvin resistance $R_{Sh} = (e^2 k_F^2 A/4\pi^2 \hbar)^{-1}$, and A is the area of the junction.

Another possibility is that X is a dirty normal wire of conductance G_N and an insulating layer with conductance G_T crosses the wire [this insulating layer, for example, can be situated at the S_F-X interface]. In this case the distribution of the transmission eigenvalues $\rho(\mathcal{T})$ is known [22]; for example, if $G_T/G_N \gg 1$, then $\rho(\mathcal{T}) = (\pi \hbar G_N/e^2)/\mathcal{T}\sqrt{1-\mathcal{T}}$ [23]. The graph of the critical current versus the exchange field is shown in the Fig. 3b for a set of values of $\alpha \equiv G_T/G_N$. It follows from this figure that in the metallic regime, $\alpha \gg 1$, when both small and large transmission eigenvalues give the main contribution to the current, the EFSE is suppressed. If X consists of two insulating barriers separated by a dirty normal wire, $\rho \propto 1/\mathcal{T}^{3/2}\sqrt{1-\mathcal{T}}$; there is a weak EFSE-effect in this case, the relative supercurrent enhancement does not exceed 10%.
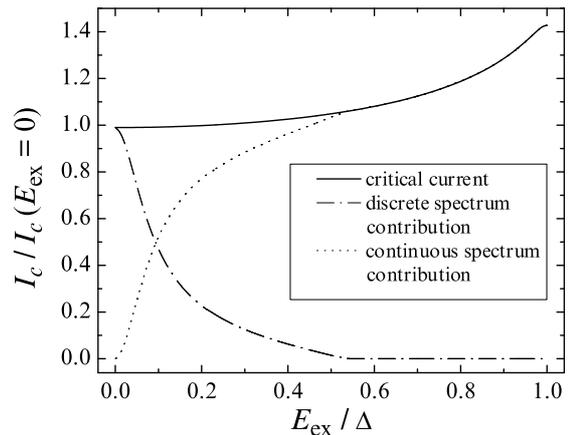

Fig. 4 The critical current in an $S_F X S_F$ junction with $\Delta_1 = \Delta_2$, $E_{ex} \equiv h_1 = -h_2$, $\rho(\mathcal{T}) \propto \delta(\mathcal{T} - D)$, and $D = 0.2$. The figure shows the relative contributions to the critical current of the discrete spectrum (Andreev levels) and the continuous spectrum.

Figure 4 shows the relative contribution of the discrete spectrum (Andreev levels) and the continuous spectrum to the supercurrent. It turns out that the EFSE-effect is mostly due to the continuous spectrum: the contribution of the discrete spectrum to the supercurrent decreases with the exchange field, while the contribution of the continuous spectrum increases. If X is an insulator, the continuous spectrum gives the main contribution to the supercurrent (see Fig. 2) and there is a pronounced EFSE effect.

Finally we discuss the AC Josephson effect in $S_F I S_F$ structures. Similarly as in tunnel SIS junctions [10–12], the current consists of three parts: $I(t) = I_1(t) + I_2(t) + I_3$, where $I_1(t) = \mathrm{Re}[I_c(V,h)]\sin(2eVt/\hbar)$ is the supercurrent, $I_2(t) = \mathrm{Im}[I_c(V,h)]\cos(2eVt/\hbar)$ the interference current, and I_3 the quasiparticle current, here, $h = h_1 - h_2$. Here, we concentrate on the behavior of I_1 and I_2; the quasiparticle current has been studied in [13]. The complex supercurrent amplitude $I_c(V,h)$ in an $S_F I S_F$ junction can be calculated in a similar way as in an SIS junction [10, 11]. At zero temperature, it has the remarkable property that

$$I_c(V,h) = \frac{1}{2}(I_c(V + h/e, 0) + I_c(V - h/e, 0)). \quad (12)$$

By setting $V = 0$, we find again that the DC critical current of an $S_F I S_F$ junction coincides with the real part of the AC supercurrent amplitude of an SIS junction if we replace eV by h. Using Eq. (12) we can also discuss the AC Josephson effect of the $S_F I S_F$ junction. In an SIS junction, $\mathrm{Re}\, I_c(V)$ has a Riedel singularity at $|eV| = \Delta_1 + \Delta_2$; but in the $S_F I S_F$ case, the Riedel singu-
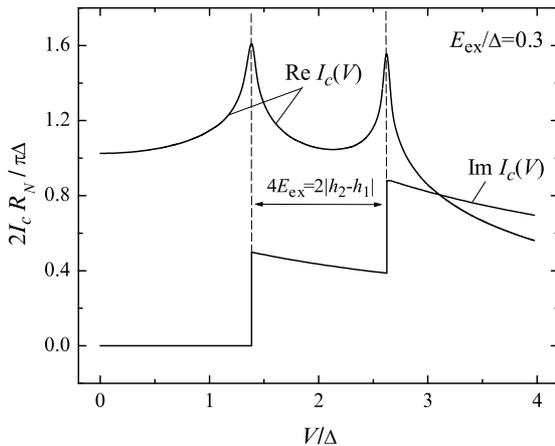
Fig. 5 Real and imaginary parts of the AC Josephson supercurrent amplitude $I_c(V)$ in an $S_F I S_F$ junction at $T = 0$, $E_{\mathrm{ex}} \equiv h_1 = -h_2 = 0.3\Delta$, and $\Delta_1 = \Delta_2 \equiv \Delta$. Riedel-type singularities are visible at $V = 2\Delta \pm 2E_{\mathrm{ex}}$.

larity appears at $|eV \pm (h_2 - h_1)| = \Delta_1 + \Delta_2$ (we assume a collinear orientation of the exchange fields $h_{1,2}$). In an SIS junction, $\mathrm{Im}\, I_c(V)$ vanishes for $|eV| < \Delta_1 + \Delta_2$, and jumps to $\pi\sqrt{\Delta_1 \Delta_2}/2R_N$ at $|eV| = \Delta_1 + \Delta_2$ [12]. In contrast, in an $S_F I S_F$ junction, $\mathrm{Im}\, I_c(V)$ jumps at $|eV - (h_2 - h_1)| = \Delta_1 + \Delta_2$ [see Fig. 5], and the the jump is half as big as in the SIS case.

In conclusion, we have shown that there is a pronounced exchange field supercurrent enhancement effect in $S_F X S_F$ junctions if the distribution of transmission eigenvalues of the X-layer has maximum weight at small values. If X is a diffusive normal metal, there is no exchange field enhancement of the supercurrent. At small temperatures, there is a correspondence between the critical current in an $S_F I S_F$ junction with collinear orientations of the exchange fields and the supercurrent amplitude in an SIS tunnel junction in the AC regime: the difference of the exchange fields in an $S_F I S_F$ junction is the analogue of the voltage in an SIS junction. Finally, we also discussed the AC Josephson effect in $S_F I S_F$ junctions.

---

1. G. Sarma, J. Phys. Chem. Solids **24**, 1029 (1963); see also D. Saint-James, G. Sarma, and E. J. Thomas, *Type II Superconductivity* (Pergamon, Oxford, 1969), p. 159.
2. A. I. Buzdin, L. N. Bulaevskiǐ, M. L. Kulic *et al.*, Soviet Phys. Uspekhi **27**, 927 (1984).
3. Ya. V. Fominov, N. M. Chtchelkatchev, and A. A. Golubov, Pis'ma ZhETF **74**, 101 (2001) [JETP Lett **74**, 96 (2001)].
4. E. A. Demler, G. B. Arnold, and M. R. Beasley, Phys. Rev. B **55**, 15174 (1997).
5. F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. Lett **86**, 3140 (2001).
6. V. N. Krivoruchko and E. A. Koshina, cond-mat/0104251.
7. N. M. Chtchelkatchev, W. Belzig, Yu. V. Nazarov, and C. Bruder, Pis'ma Zh. Eksp. Teor. Fiz. **74**, 357 (2001) [JETP Lett. **74**, 323 (2001)].
8. A. A. Golubov, M. Yu. Kupriyanov, and Ya. V. Fominov, Pis'ma Zh. Eksp. Teor. Fiz. **75**, 223 (2002) [JETP Lett. **75**, 190 (2002)].
9. E. Riedel, Z. Naturforsch. **19a**, 1634 (1964).
10. N. R. Werthamer, Phys. Rev. **147**, 255 (1966).
11. A. I. Larkin and Yu. N. Ovchinnikov, Sov. Phys. JETP **24**, 1035 (1967).
12. I. O. Kulik and I. K. Yanson, *The Josephson Effect in Superconductive Tunneling Structures* (Israel Program for Scientific Translations, Jerusalem, 1972) [Nauka, Moscow, 1970].
13. D. H. Huertas-Hernando, Yu. V. Nazarov, and W. Belzig, Phys. Rev. Lett. **88**, 047003 (2002).
14. A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1986).
15. Ya. V. Fominov, N. M. Chtchelkatchev, and A. A. Golubov, cond-mat/0202280.
16. A. V. Zaitsev, Sov. Phys. JETP **59**, 1163 (1984).
17. Yu. V. Nazarov, Superlattices and Microst. **25**, 1221 (1999).
18. M. Yu. Kupriyanov and V. F. Lukichev, Zh. Exp. Teor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].
19. W. Belzig, F. K. Wilhelm, C. Bruder *et al.*, Superlattices and Microst. **25**, 1251 (1999).
20. C. W. J. Beenakker, Phys. Rev. Lett. **67**, 3836 (1991).
21. I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products* (Academic Press, New York, 1980).
22. Yu. V. Nazarov, Phys. Rev. Lett. **73**, 134 (1994).
23. O. N. Dorokhov, Solid State Commun. **51**, 381 (1984).