Strategy for implementing stabilizer-based codes on solid-state qubits

Tetsufumi Tanamoto,[1] Vladimir M. Stojanović,[2] Christoph Bruder,[2] and Daniel Becker[2]

[1] *Corporate R & D center, Toshiba Corporation, Saiwai-ku, Kawasaki 212-8582, Japan*

[2] *Department of Physics, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland*

(Dated: January 18, 2013)

We present a method for implementing stabilizer-based codes with encoding schemes of the operator quantum error correction paradigm, e.g., the "standard" five-qubit and CSS codes, on solid-state qubits with Ising or XY-type interactions. Using pulse sequences, we show how to induce the effective dynamics of the stabilizer Hamiltonian, the sum of an appropriate set of stabilizer operators for a given code. Within this approach, the encoded states (ground states of the stabilizer Hamiltonian) can be prepared without measurements and preserved against both the time evolution governed by the original qubit Hamiltonian, and energy-nonconserving errors caused by the environment.

PACS numbers: 03.67.Lx,03.67.Pp,03.67.Ac

A variety of quantum error-correcting codes (QECC) have been widely investigated aiming at a robust computing system similar to the classical digital computer [1–14]. In particular, codes based on the stabilizer formalism constitute an important class of QECCs. This formalism has proven useful not only for the standard codes [1, 2], but also for the subsystem code [4–6], topological [3, 7, 15], and Majorana codes [8]. On the experimental side, Knill *et al.* demonstrated its usefulness in the NMR domain [10, 11]. Stabilizer-based QECC in systems with always-on coupling have recently attracted a great deal of interest [16, 17].

Stabilizer operators G_j ($j = 1, \ldots, l$) are mutually commuting operators given by products of multiple Pauli matrices X_i, Y_i, and Z_i ($i = 1, \ldots, n$) [2]. Conventionally, logical qubit states are encoded through measurements into a joint, 2^l-dimensional, eigenspace $\mathcal{H}_S$ of these operators. For l stabilizer operators and n physical qubits, a maximum number of $k = n - l$ logical qubits can be encoded into $\mathcal{H}_S$, while $k < n - l$ in case of subsystem encoding. Since the ground states of the stabilizer Hamiltonian $H_{\rm stab} := -\sum_{j=1}^{l} G_j$ are joint eigenstates of all stabilizer operators, its ground state manifold can play the role of $\mathcal{H}_S$.

It is important to note that stabilizer operators of many error-correction codes, e.g., the surface code [3] or color code [7], are given by products of more than two Pauli matrices. Therefore the corresponding stabilizer Hamiltonians cannot be directly implemented in natural solid-state qubit systems, where the interactions between qubits are of two-body type [18, 19].

In this letter, we demonstrate how to prepare ground states of $H_{\rm stab}$ as encoded states and preserve them by inducing the effective dynamics of this Hamiltonian using sequences of pulses in the form of single-qubit rotations. Being based on single-qubit rotations only, our method works for an always-on physical (qubit) Hamiltonian, i.e., it does not require switching on and off any of its parts (single-qubit or interaction).

The distinguishing feature of this method is that it allows the preparation of encoded states without measurements, thus avoiding measurement-induced decoherence. The method can be used not only for standard codes (i.e., five-qubit and CSS codes) but also for the extended class of codes with encoding schemes within the general operator quantum error correction framework [4, 5]. Even in the presence of inevitable pulse (rotation angle) errors the ground-state fidelity scales favorably with the system size.

Our scheme allows the implementation of stable solid-state quantum memories. This, in turn, facilitates the realization of quantum gates [17] within the limitations imposed by the size and coherence time of a system. Although the scheme requires a rather large number of pulses (rotations), its feasibility can be anticipated based on the recent progress in qubit-manipulation techniques [20].

Generation of stabilizer operators.— We consider the Hamiltonian $H = H_0 + H_{\rm XY}$, where $H_0 = \sum_i H_{0i} = \sum_i(\Omega_i X_i + \varepsilon_i Z_i)$ is a single-qubit part and $H_{\rm XY} = \sum_i H_{XY}^{i,i+1} \equiv \sum_i J_{i,i+1}(X_i X_{i+1} + Y_i Y_{i+1})$ the (two-body) XY-interaction part. Note that instead of the XY-Hamiltonian we could also use the Ising Hamiltonian $H_{\rm Ising} = \sum_i J_{i,i+1} Z_i Z_{i+1}$.

Higher-order products of Pauli matrices can be generated using the following transformations [21]:

$$\begin{aligned} e^{-itH_{XY}^{i,i+1}} X_i e^{itH_{XY}^{i,i+1}} &= c_\theta X_i - s_\theta Z_i Y_{i+1} \,, \\ e^{-itH_{XY}^{i,i+1}} Y_i e^{itH_{XY}^{i,i+1}} &= c_\theta Y_i + s_\theta Z_i X_{i+1} \,, \\ e^{-itH_{XY}^{i,i+1}} Z_i e^{itH_{XY}^{i,i+1}} &= c_\theta^2 Z_i + s_\theta^2 Z_{i+1} \\ &+ c_\theta s_\theta (X_i Y_{i+1} - Y_i X_{i+1}) \,, \quad (1) \end{aligned}$$

where $c_\theta \equiv \cos(2\theta)$ and $s_\theta \equiv \sin(2\theta)$. For $\theta = Jt = \pi/4$, these transformations increase the order of the Pauli-matrix terms as $X_i \to -Z_i Y_{i+1}$ and $Y_i \to Z_i X_{i+1}$. Similarly, one obtains $Z_i \to Z_{i+1}$. Analogous relations hold for the Ising interaction.

Manipulation of the Hamiltonian.— The key step in obtaining the local stabilizer operator is extracting a

		$H_{\rm ini}$	process : $e^{-i\tau_{\rm op}H_{\rm op}}He^{i\tau_{\rm op}H_{\rm op}}$	time for generation
G_1	$X\ Z\ Z\ X\ I$	Ω_2X_2	$H_{XY}^{23}\to(\pi/2)_2^x\to H_{XY}^{12}+H_{XY}^{34}$	$\tau_{\rm ini}+24\tau_{\rm rot}+4\tau_{\rm op}$
G_2	$I\ X\ Z\ Z\ X$	Ω_3X_3	$H_{XY}^{34}\to(\pi/2)_3^x\to H_{XY}^{23}+H_{XY}^{45}$	$\tau_{\rm ini}+24\tau_{\rm rot}+4\tau_{\rm op}$
G_3	$X\ I\ X\ Z\ Z$	Ω_2X_2	$G_1\to H_{XY}^{45}\to(\pi/2)_2^x(\pi/2)_5^x\to H_{XY}^{23}$	$\tau_{\rm ini}+43\tau_{\rm rot}+8\tau_{\rm op}$
G_4	$Z\ X\ I\ X\ Z$	Ω_2X_2	$G_1\to H_{XY}^{45}\to(\pi/2)_3^x(\pi/2)_5^x\to H_{XY}^{23}\to(\pi/2)_1^y(\pi/2)_4^y$	$\tau_{\rm ini}+45\tau_{\rm rot}+8\tau_{\rm op}$

TABLE I: Stabilizer operators for the five-qubit code [1] as realized with XY-type interactions. Each appearance of $H_{XY}^{i,i+1}$ indicates an application of the transformation $H\to\ e^{-i\tau_{\rm op}H_{\rm op}}He^{i\tau_{\rm op}H_{\rm op}}$, while each instance of $(\pi/2)_i^x$ denotes a $\pi/2$ rotation about the x-axis. The rightmost column shows the time required to generate each stabilizer operator; $\tau_{\rm rot}$ is the time needed to perform a single-qubit rotation.

single-qubit part or a pure two-body interaction part from the original system Hamiltonian. This process is carried out using the Baker-Campbell-Hausdorff (BCH) formula [22]. For simplicity, we explain this procedure for $\varepsilon_i=0$ in H_0, where only rotations about the z axis will be needed, and set $\Omega_i=\Omega$. In the general case, the procedure requires a slightly more complex pulse sequence.

A part H_a can be extracted from H_0 by applying a single appropriate π pulse, if that pulse transforms H_0 to H_a-H_b, where $H_b=H_0-H_a$ consists of the unwanted terms. For $2n$ alternating periods of propagation with $A=i\tau(H_a+H_b)$ and $B=i\tau(H_a-H_b)$, the BCH formula yields

$$(e^Ae^B)^n\approx\exp(i2n\tau H_a+n\tau^2[H_a,H_b]) \qquad (2)$$

where the duration of the pulse sequence is $2n\tau$. Thus, as long as $\tau||H_b||\ll 1$, where $||A||=[{\rm Tr}(A^\dagger A)/d]^{1/2}$ is the standard operator norm in a Hilbert space of dimension d, we can neglect the second term. As the number n of repetitions increases, this approximation becomes progressively better.

In order to extract a single-qubit (local) part of the system Hamiltonian, relation (2) has to be applied twice, leading to (case $n=1$)

$$\begin{aligned} e^Ae^Be^{B'}e^{A'} &\approx \exp(2h_a+[h_b,h_a])\exp(2h'_a-[h'_b,h'_a]) \\ &\approx \exp(2(h_a+h'_a)+[h_b,h_a]-[h'_b,h'_a]+4[h_a,h'_a])\end{aligned} \qquad (3)$$

where $h_{a/b}^{(\prime)}:=i\tau H_{a/b}^{(\prime)}$, $A'=h'_a+h'_b$ and $B'=h'_a-h'_b$. Consider extracting a single-qubit Hamiltonian X_2 for a one-dimensional five-qubit array. With $h_i=i\tau\Omega X_i$ (single-qubit Hamiltonian with $\epsilon_i=0$) and $h_{ij}=i\tau H_{XY}^{ij}$, we set

$$\begin{aligned} h_a &= h_2+h_{34}+h_{45}, \\ h_b &= h_1+h_3+h_4+h_5+h_{12}+h_{23}, \\ h'_a &= h_2-h_{34}-h_{45}, \\ h'_b &= h_1-h_3+h_4-h_5+h_{12}-h_{23}. \end{aligned} \qquad (4)$$

The sequence of operators describing the time evolution on the left-hand side of Eq. (3) is obtained in the following manner: by applying a π-pulse to qubits 1, 3, 4, and 5 one transforms A into B and B' into A', while B is transformed into B' by a π-pulse applied to qubits 3 and 5. This leads to $H_{\rm ini}=H_{\rm ini}^{(0)}+H_{\rm ini}^{(1)}$, where $H_{\rm ini}^{(0)}=\Omega X_2$ is the desired initial Hamiltonian and $H_{\rm ini}^{(1)}=[\Omega\tau/2](J_{32}Y_3Z_2-J_{34}Y_3Z_4-J_{45}Y_5Z_4)$ is an unwanted perturbation term. This term scales like $J_{ij}\tau\ll 1$ and hence can be reduced by shortening the duration of the pulse sequence.

Similarly, the operator $e^{-itH_{XY}^{12}}$ is obtained by extracting H_{XY}^{12} from the system Hamiltonian using

$$\begin{aligned} h_a &= h_{23}+h_1+h_4, \\ h_b &= h_{12}+h_{34}+h_{45}+h_2+h_3+h_5, \\ h'_a &= h_{23}-h_1-h_4, \\ h'_b &= -h_{12}-h_{34}-h_{45}+h_2+h_3+h_5. \end{aligned} \qquad (5)$$

The perturbation terms can be neglected for $J/\Omega\ll 1$.

Time-evolution of the system— To illustrate the time evolution of the system during the generation process for the stabilizer operator G_j we will use the schematic notation $\rho(0)\xrightarrow{tH}\rho(t)$. Here $\rho(t)=\exp(-iHt)\rho(0)\exp(iHt)$ is the density matrix for a time-independent Hamiltonian H, or for an effective H in the sense of average-Hamiltonian theory [22]. The manipulation of the original Hamiltonian $H_{\rm ini}$ is described by [21] $\rho(0)\xrightarrow{\tau_{\rm op}H_{\rm op}}\xrightarrow{\tau_{\rm ini}H_{\rm ini}}\xrightarrow{-\tau_{\rm op}H_{\rm op}}\rho(t)$, where $\tau_{\rm op}=\pi/(4J)$ and $H_{\rm op}$ is either the pairwise interaction H_{XY}^{ij} or a generator of a single-qubit rotation, see Table I. During this pulse sequence the stabilizer operator G_j describes the dynamics of the system in the sense of average-Hamiltonian theory [21]. Here $\tau_{\rm ini}$ has to be chosen such that the entire process can be carried out in a time interval sufficiently shorter than the coherence time.

In the following we apply our scheme to the standard codes (five-qubit and CSS codes) [1], as well as the surface code [3].

Realization of the five-qubit and CSS codes.— The generation processes of the four stabilizer operators $G_j(j=1,\ldots,4)$ of the five-qubit code [1] are shown in Table I. For example, starting from the initial Hamiltonian $H_{\rm ini}=\Omega_2X_2$, the stabilizer operator G_1 of the five-qubit code is realized through the sequence

$$\begin{aligned} e^{-i\tau_{\rm op}H^{23}_{XY}}X_2e^{i\tau_{\rm op}H^{23}_{XY}} &\to -Z_2Y_3 \\ -e^{-i(\pi/4)X_2}Z_2Y_3e^{i(\pi/4)X_2} &\to Y_2Y_3 \\ e^{-i\tau_{\rm op}[H^{12}_{XY}+H^{34}_{XY}]}Y_2Y_3e^{i\tau_{\rm op}[H^{12}_{XY}+H^{34}_{XY}]} &\to X_1Z_2Z_3X_4\,. \end{aligned} \quad (6)$$

The minimal time required for this process is $\tau_{\rm ini} + 24\tau_{\rm rot} + 4\tau_{\rm op}$. The effective dynamics of $H_{\rm stab} = -\sum_{l=1}^{4} G_l$ is induced by subsequent generation of the four stabilizer operators.

We would now like to address the feasibility of this scheme in a typical superconducting qubit system. For two superconducting qubits in a circuit-QED setup the resulting effective inter-qubit interaction is also of XY type [23, 24]. For instance, for $g/\Delta = 0.1$, $g/(2\pi) = 200$ MHz, $\Delta/(2\pi) = 2$ GHz, where g is the Jaynes-Cummings coupling constant and Δ the detuning between the resonator frequency and the qubit splitting, we have $J/(2\pi) = 20$ MHz. Assuming $\tau_{\rm rot} \sim 1$ ns [25], we obtain a minimal total time of $\tau^{\rm min}_{\rm 5code} = 24\tau_{\rm op} + 136\tau_{\rm rot} \approx 300$ ns, which is significantly shorter than $T_2 \sim 20$ μs reported in [26].

Table II shows how to generate the CSS code. The stabilizer operators G_4, G_5, G_6 are obtained by $e^{-\pi\sum_i Y_i/4}(G_1 + G_2 + G_3)e^{\pi\sum_i Y_i/4}$. Thus, the minimal total time is $\tau^{\rm min}_{\rm CSS} = 44\tau_{\rm op} + 246\tau_{\rm rot} \approx 600$ ns, i.e., again much shorter than the T_2 given in Ref. [26].

Realization of the surface code.— Four types of stabilizer operators are needed to realize the Hamiltonian of this code. Qubits are placed at the edges of the square lattice, see Fig. 1. Stabilizer operators $H_s = \Pi_{j\in {\rm star(s)}}X_j$ are assigned to each vertex s, and $H_p = \Pi_{j\in {\rm boundary}(p)}Z_j$ to each face p. Using the relations $X_i \to -Z_iY_{i+1}$ and $Y_i \to Z_iX_{i+1}$, we can form products of nearest-neighbor operators such as $Y_1 \to Z_1X_2 \to -Z_1Z_2Y_3 \to Z_1Z_2Z_3X_4$. In generating adjacent stabilizer operators, care should be taken to avoid mixing them. This can be achieved by decompositions like $H_s = H_{s_1} + H_{s_2}$ [see Fig. 1(a) and (b)] and $H_p = H_{p_1} + H_{p_2}$ [Fig. 1(c) and (d)]. The four elements can then be combined into the total surface-code Hamiltonian.

Preparation of encoded states— Our approach also allows us to prepare encoded states (or codewords) of general stabilizer-based codes without performing measurements on the system. We show this in detail for the standard codes, which encode k logical qubits into a subspace of dimension 2^k. However, this procedure also works for subsystem encoding provided suitable stabilizer operators are added. For any given code, only those G_j with $1 \le j \le m$ and $m \le n-k$ that contain X or Y operators are needed for the preparation:

$$\begin{aligned} |\bar c_1...\bar c_k\rangle &= (1+G_1)\cdots(1+G_m)\bar X_1^{c_1}\cdots\bar X_k^{c_k}|0...0\rangle \\ &= \prod_{i=1}^{k}\bar X_i^{c_i}\prod_{j=1}^{m}\exp\left(i\frac{\pi}{4}\tilde G_j\right)|0...0\rangle\,, \end{aligned} \quad (7)$$

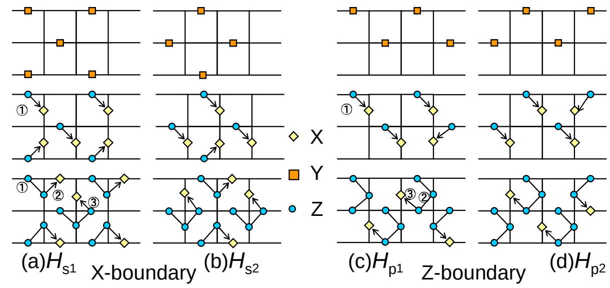

FIG. 1: Four types of surface-code generation on the 2×3 lattice in Ref. [3] starting from a single-qubit Hamiltonian that includes Y operators. All operators in (a), (b) are transformed to X and those in (c), (d) to Z. Subsequently, the four types of stabilizer operators are combined into the topological Hamiltonian in the manner discussed in the text.

where $c_i = 0, 1$ and operators $\bar X_i$ act in the logical state space $\{|\bar 0\rangle_i, |\bar 1\rangle_i\}$. Here, $\tilde G_j$ denotes a modified stabilizer operator obtained from G_j by replacing one of the X operators—acting on, say, qubit a—by a Y operator, or vice versa. This is done in order to match the effect of $\exp[i(\pi/4)\tilde G_j]$ with the action of the projector $(1 + G_j)$ when qubit a is in state $|0\rangle$.

We illustrate the encoding procedure on the example of a three-qubit code whose stabilizer operators are X_1X_2 and X_2X_3. This is realized in a three-qubit system with Ising interactions. The stabilizer Hamiltonian $J(X_1X_2 + X_2X_3)$ is obtained by removing the single-qubit part H_0 of the original Hamiltonian using π-pulses. Its ground states are the degenerate states $|\bar 0\rangle = |000\rangle + |110\rangle + |011\rangle + |101\rangle$ and $|\bar 1\rangle = |001\rangle + |010\rangle + |100\rangle + |111\rangle$. They are obtained by $|0\rangle_L = \frac{1}{2}(1 + X_1X_2)(1 + X_2X_3)|000\rangle = \exp[i(\pi/4)Y_1X_2]\exp[i(\pi/4)X_2Y_3]|000\rangle$.

For the five-qubit code, we have $\tilde G_1 = Y_1Z_2Z_3X_4$, $\tilde G_2 = X_2Z_3Z_4Y_5$, $\tilde G_3 = X_1Y_3Z_4Z_5$, $\tilde G_4 = Z_1Y_2X_4Z_5$, and the multiplication in Eq. (7) is carried out in the following order: $\exp[i(\pi/4)\tilde G_2]\exp[i(\pi/4)\tilde G_4]\exp[i(\pi/4)\tilde G_3]$ $\exp[i(\pi/4)\tilde G_1]$. In the case of the CSS code, $\tilde G_1 = X_1X_2X_3Y_4$, $\tilde G_2 = X_1X_2X_5Y_6$, $\tilde G_3 = X_1X_3X_5Y_7$. Note that here only three out of six stabilizer operators are needed for the preparation of an encoded state.

Robustness against pulse errors.— Since the codeword states are encoded in the twofold-degenerate ground-state manifold $|\bar 0\rangle$ and $|\bar 1\rangle$ of $H_{\rm stab}$, the robustness of this method is limited by the rate of leakage out of this manifold. In principle, precise estimates of the leakage due to the thermal environment could be obtained by studying the stability of the ground state to various perturbations as in Ref. [27]. However, already single-qubit errors—in most cases the prevalent kind of errors created by a thermal bath—are suppressed by a factor of $e^{-2\Omega/T}$ or smaller for the five-qubit and CSS code [28]. Hence, at low temperatures, unavoidable pulse imperfections are likely to be the predominant cause of leakage.

To estimate this effect, we consider pulse errors that

		$H_{\rm ini}$	process : $e^{-i\tau_{\rm op}H_{\rm op}}He^{i\tau_{\rm op}H_{\rm op}}$	time for generation
G_1	$X\ X\ X\ X\ I\ I\ I$	$\Omega_2 X_2$	$H_{XY}^{23}\to(\pi/2)_2^x\to H_{XY}^{12}+H_{XY}^{34}\to(\pi/2)_2^x(\pi/2)_3^x$	$\tau_{\rm ini}+26\tau_{\rm rot}+4\tau_{\rm op}$
G_2	$X\ X\ I\ I\ X\ X\ I$	$-\Omega_3 X_3$	$H_{XY}^{34}\to(\pi/2)_3^y\to H_{XY}^{23}+H_{XY}^{45}\to H_{XY}^{12}+H_{XY}^{56}\to(\pi/2)_3^x(\pi/2)_4^x(\pi/2)_6^x\to H_{XY}^{23}+H_{XY}^{45}\to(\pi/2)_6^y$	$\tau_{\rm ini}+45\tau_{\rm rot}+8\tau_{\rm op}$
G_3	$X\ I\ X\ I\ X\ I\ X$	$-\Omega_3 X_3$	$H_{XY}^{34}\to(\pi/2)_3^x\to H_{XY}^{23}+H_{XY}^{45}\to H_{XY}^{12}+H_{XY}^{56}\to H_{XY}^{67}\to(\pi/2)_1^x(\pi/2)_2^x(\pi/2)_4^x(\pi/2)_6^x\to H_{XY}^{12}+H_{XY}^{34}+H_{XY}^{56}$	$\tau_{\rm ini}+51\tau_{\rm rot}+10\tau_{\rm op}$

TABLE II: Stabilizer operators for the CSS code [1] as realized with XY-type interactions. The operators G_4, G_5, and G_6 are obtained by replacing X with Z in G_1, G_2, and G_3, respectively. The rightmost column shows the time required to generate each stabilizer operator; $\tau_{\rm rot}$ is the time needed to perform a single-qubit rotation.

can be modeled by randomly distributed, unbiased, and uncorrelated deviations $\delta\theta$ with $\sigma_\theta=\sqrt{\langle\delta\theta^2\rangle}$ from the ideal angle of $\pi/2$. The leakage can then be estimated by looking at the average of the ground state fidelity $F(t)=|\langle\bar{0}|U_{\rm P}(t)|\bar{0}\rangle|^2$, where $U_{\rm P}(t)$ is the time evolution operator with imperfect pulses. This average is approximately given by $\langle F(t)\rangle\approx 1-N_{\rm P}\sigma_\theta^2 t/(8\mathcal{T})$, where $N_{\rm P}$ is the number of pulses in the sequence to generate $H_{\rm stab}$, and $\mathcal{T}$ its duration. The number $N_{\rm P}$ of pulses is given by the number of rotations needed to generate all stabilizers of a given code (for the five-qubit and CSS code, see Table I and Table II, respectively).

In conclusion, we have demonstrated measurement-free preparation of encoded states in the stabilizer-based codes described by the operator quantum error correction paradigm. The scheme is based on pulse sequences applied to solid-state qubit Hamiltonians with two-body interactions of XY- or Ising-type. We have estimated the intrinsic robustness of our scheme against pulse imperfections. Once implemented experimentally, our scheme will pave the way for robust quantum information processing.

We would like to thank Fabio Pedrocchi and Adrian Hutter for discussions. This work was financially supported the Swiss SNF and the NCCR Quantum Science and Technology.

[1] D. Gottesman, quant-ph/9705052.
[2] M.A. Nielsen and I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[3] A. Kitaev, Ann. Phys. (NY) **303**, 2 (2003); E. Dennis, A. Kitaev, A. Landahl, and J. Preskill, J. Math. Phys. **43**, 4452 (2002).
[4] D. Kribs, R. Laflamme, and D. Poulin, Phys. Rev. Lett. **94**, 180501 (2005).
[5] D. Poulin, Phys. Rev. Lett. **95**, 230504 (2005).
[6] D. Bacon, Phys. Rev. A **73**, 012340 (2006).
[7] H. Bombin and M.A. Martin-Delgado, Phys. Rev. B **75**, 075103 (2007).
[8] S. Bravyi, B.M. Terhal, and B. Leemhuis, New J. Phys. **12**, 083039 (2010).
[9] P. Milman, W. Maineult, S. Guibal, L. Guidoni, B. Doucot, L. Ioffe, and T. Coudreau, Phys. Rev. Lett. **99**, 020503 (2007).
[10] E. Knill, R. Laflamme, R. Martinez, and C. Negrevergne, Phys. Rev. Lett. **86**, 5811 (2001).
[11] J. Chiaverini, D. Leibfried, T. Schaetz, M. D. Barrett, R.B. Blakestad, J. Britton, W.M. Itano, J. D. Jost, E. Knill, C. Langer, R. Ozeri, and D.J. Wineland, Nature (London) **432**, 602 (2004).
[12] M. D. Reed, L. DiCarlo, S. E. Nigg, L. Sun, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **482**, 382 (2012).
[13] O. Moussa, J. Baugh, C.A. Ryan, and R. Laflamme, Phys. Rev. Lett. **107**, 160501 (2011).
[14] K. Keane and A.N. Korotkov, Phys. Rev. A **86**, 012333 (2012).
[15] J. R. Wootton and D. Loss, Phys. Rev. Lett. **109**, 160503 (2012).
[16] S. E. Nigg and S. M. Girvin, arXiv:1212.4000.
[17] A. De and L. P. Pryadko, arXiv:1209.2764.
[18] T. Tanamoto, Y.X. Liu, X. Hu, and F. Nori, Phys. Rev. Lett. **102**, 100501 (2009).
[19] Os Vy, Xiaoting Wang, and K. Jacobs, arXiv:1210.8007.
[20] I. Buluta, S. Ashhab, and F. Nori, Rep. Prog. Phys. **74**, 104401 (2011).
[21] T. Tanamoto, D. Becker, V.M. Stojanović, and C. Bruder, Phys. Rev. A **86**, 032327 (2012).
[22] R.R. Ernst, G. Bodenhausen, and A. Wokaun, *Principles of Nuclear Magnetic Resonance in One and Two Dimensions* (Oxford University Press, Oxford, 1987).
[23] A. Blais, R.-S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **69**, 062320 (2004).
[24] V.M. Stojanović, A. Fedorov, A. Wallraff, and C. Bruder, Phys. Rev. B **85**, 054504 (2012).
[25] J. M. Chow *et al.*, Phys. Rev. A **82**, 040305 (2010).
[26] Hanhee Paik, D.I. Schuster, L.S. Bishop, G. Kirchmair, G. Catelani, A.P. Sears, B.R. Johnson, M.J. Reagor, L. Frunzio, L.I. Glazman, S.M. Girvin, M.H. Devoret, and R.J. Schoelkopf, Phys. Rev. Lett. **107**, 240501 (2011).
[27] S. Bravyi, M. Hastings, and S. Michalakis, J. Math. Phys. **51**, 093512 (2010).
[28] This is actually the case for any code that is protected against arbitrary single-qubit errors and can be implemented with pulses.