Visibility of the Aharonov-Bohm Effect in a Ring Coupled to a Fluctuating Magnetic Flux*

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We consider the visibility of the Aharonov-Bohm effect for cotunneling transport through a clean one-channel ring coupled to a fluctuating magnetic flux. We concentrate on the modification of the destructive interference at $\Phi_0/2$ *by the fluctuating flux, since changes in the magnitude of the current away from this point can also be caused by renormalization effects and do not necessarily indicate dephasing. For fluctuations arising from the Nyquist noise in an external coil at* $T = 0$*, the suppression of the destructive interference shows up only in a contribution proportional to* V^3*, and therefore does not affect the linear conductance. In this sense, the Nyquist bath does not lead to dephasing in the linear transport regime at zero temperature in our model.*

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Recently, the question of dephasing in diffusive interacting electronic systems at low temperatures has attracted a lot of attention and controversy (see e.g. Ref. 1). In this paper, we will discuss a simpler model problem: the influence of an external bath on the Aharonov-Bohm (AB) effect, i.e. on the magnetic flux dependence of the current through a ring. Various cases of this problem have been treated in the past[2–10]. In the present work, the bath will be provided by a fluctuating magnetic flux (which may be due to Nyquist noise in an external coil), and we will focus on the *suppression of destructive interference* at $\Phi = \Phi_0/2$. As a result of our calculation, we find that the fluctuations do *not* lead to dephasing in the *linear* transport regime at *zero* temperature.

The aim of the presentation is to make contact with the Feynman-Vernon influence functional formalism which is often used to discuss de-

*dedicated to Peter Wölfle on the occasion of his 60th birthday.

phasing in a single-particle picture. Its direct applicability is ruled out in a low-temperature situation where the restrictions owing to the Pauli principle become important. Some authors have tried to account for this fact by dropping certain zero-point contributions "by hand", thereby extending results of a semiclassical dephasing calculation to lower temperatures[11]. The model system presented here allows to observe directly the modifications brought about by both energy conservation and the Pauli principle at low temperatures (and low bias voltages), in a particularly simple situation. Although the model is not directly related to the above-mentioned puzzling questions concerning the suppression of weak localization effects due to coupling between the electrons and a bath, we hope that it helps to clarify these points.

In our model, the ring is coupled to two electrodes by tunnel contacts and threaded by a fluctuating magnetic flux, see Fig. 1. We consider a Coulomb blockade situation, in which any electron tunneling into (or out of) the ring will enhance the total energy by the charging energy of the ring and the latter is much larger than the bias voltage V and the temperature T. Therefore, transport through the ring is possible only via cotunneling, i.e. a two-step process involving a virtual intermediate state belonging to a different number of electrons on the ring[12]. A strong dependence of the tunneling current on the external magnetic flux, with a complete suppression at $\Phi_0/2$ due to destructive AB interference, will be visible only in the *elastic* cotunneling contribution, in which the electronic state of the ring is left unaltered in the tunneling process. It leads to a current linear in the applied bias voltage and will dominate the contribution stemming from inelastic electronic processes at low temperatures and for small bias voltages.

The fluctuations couple to the electrons via the vector-potential term in the kinetic energy, i.e., the Hamiltonian of the electrons on the ring is given by

$$\hat{H} \equiv \sum_p \hat{\Psi}_p^\dagger \frac{\left(p - g\hat{\phi}\right)^2}{2m} \hat{\Psi}_p + \hat{H}_{bath} \,, \tag{1}$$

where $\hat{H}_{bath}$ is given by a set of uncoupled harmonic oscillators:

$$\hat{H}_{bath} \equiv \sum_{j=1}^{N_{osc}} \left\{ \frac{\hat{P}_j^2}{2M} + \frac{M\omega_j^2}{2} \hat{Q}_j^2 \right\} \,. \tag{2}$$

For simplicity, we consider spinless electrons. Here, the flux $\hat{\phi}$ is taken to be the sum of the normal coordinates of the harmonic oscillators[13,14,4]

$$\hat{\phi} \equiv \frac{1}{\sqrt{N_{osc}}} \sum_j \hat{Q}_j \,. \tag{3}$$

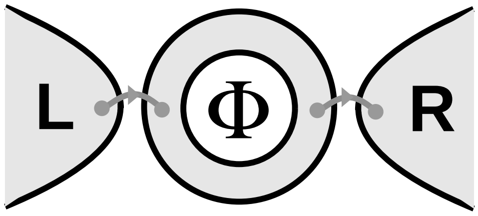

Fig. 1. The tunneling setup discussed in the text.

Any nonvanishing external static flux Φ has to be added. g is a coupling constant depending on the circumference L of the ring. The effects of the bath can be characterized fully by giving the correlator of $\hat{\phi}$, which defines the spectral function $C(\omega)$:

$$\langle \hat{\phi}(t)\hat{\phi}(0)\rangle = \int_0^\infty d\omega C(\omega)[\coth(\frac{\omega}{2T})\cos(\omega t) - i\sin(\omega t)] \; .$$

The description of the cotunneling setup is completed by adding the charging energy of electrons on the ring, the kinetic energies of electrons in the two electrodes and a tunnel coupling $\hat{H}_T$.

The tunneling process starts from a situation in which the ring is occupied by the equilibrium number of electrons (which depends on the value of an applied gate voltage) and the Fermi seas in the left and right electrode are filled up to Fermi energies that differ by the bias voltage, eV. Throughout the following discussion, we will assume the Fermi energy in the left electrode to be the larger one of the two (and $eV > 0$). In the final state, an electron has appeared above the right Fermi sea, leaving behind a hole in the left electrode. Although we want to consider the situation where the electronic state of the ring has not changed in the end, the final state of the *bath* may be different. The intermediate state is characterized by an extra electron (or extra hole) present on the ring and some arbitrary state of the bath. The tunneling rate can be obtained using Fermi's Golden rule:

$$\Gamma = 2\pi \sum_f \left| \sum_\nu \frac{H_{Tf\nu}H_{T\nu i}}{E_\nu - E_i} \right|^2 \delta(E_f - E_i) \; . \tag{4}$$

Here $\hat{H}_T = \hat{T}^L + \hat{T}^R$ is the sum of the tunneling Hamiltonians belonging to the left and the right junctions, while the energies and eigenstates refer to the unperturbed Hamiltonian that includes everything besides tunneling (in particular, it includes the coupling between electrons and the bath). We set $\hbar, k_B \equiv 1$.

Before performing the calculation in the presence of the bath, we will briefly describe how the destructive interference at $\Phi_0/2$ appears in this formula in the situation without fluctuating flux. In such a case, the intermediate state ν refers solely to the electronic state k on the ring, which is occupied by the additional electron in the course of tunneling. The final state f is determined both by the state λ, which is unoccupied in the left electrode after the tunneling process, and the state $\bar{\lambda}$, where the electron ends up in the right electrode.

The sum over intermediate states k then contains the following contribution which describes an electron going onto the ring from the left electrode and leaving through the right electrode:

$$t_L t_R^* \Psi_\lambda(y_L) \Psi_{\bar{\lambda}}^*(y_R) \sum_k \frac{\Psi_k(x_R)\Psi_k^*(x_L)}{\epsilon_k + E_C - \epsilon_\lambda} \,. \tag{5}$$

The sum over k is to be taken only over unoccupied single-electron states on the ring. For simplicity, we have assumed tunneling to take place only between two points, for example from a point y_L at the tip of the left electrode to an adjacent point x_L on the ring: $\hat{T}_L = t_L \hat{\Psi}^\dagger(x_L)\hat{\Psi}(y_L) + h.c.$, and likewise for the right electrode. t_L is a complex-valued tunneling amplitude. Such a description will be appropriate as long as the extent of the relevant region in which tunneling can take place is less than a wavelength. Ψ refers to single-electron wave functions on the ring and on the electrodes. Perfect destructive AB interference at an external static magnetic flux of $\Phi = \Phi_0/2$ arises only for an even number of electrons on the ring. In this case, the energies $\epsilon_k = (k - 2\pi\Phi/(\Phi_0 L))^2/(2m)$ of the unoccupied states are pairwise degenerate, for $k_+ \equiv n2\pi/L$ and $k_- \equiv (1-n)2\pi/L$. Therefore, the energy denominators for k_+ and k_- are the same, while the wave functions in the numerators produce a phase shift of $\exp(i(k_+ - k_-)L/2) = -1$ between the two possibilities, leading to complete cancellation of all terms in the sum.

Suppression of this perfect destructive interference is due to the electron leaving a trace in the bath that permits, at least in principle, to decide through which of the two degenerate states k_+ and k_- the electron has traveled. This involves a transfer of energy between electron and bath. The bath spectrum determines the amount of bath oscillators able to absorb the small energy $\leq eV$ which can be emitted by the electron. Therefore, one expects that dephasing at zero temperature is suppressed for $V \to 0$ due to the energy conservation constraint. This will be confirmed by the calculation described below.

After taking the modulus squared of the sum of amplitudes given above in Eq. (5), which we briefly denote by $A(k)$, one obtains "classical" probabilities like $|A(k_+)|^2$ but also cross-terms of the form $A^*(k_+)A(k_-)$. A bath coupling to the electronic motion will affect these terms differently, if it is able to "distinguish" between the momenta k_+ and k_-. Usually, this will result in a suppression of the cross-terms. Therefore, the different contributions cannot cancel any more. Away from perfect destructive interference, we have to expect an influence of the bath on the magnitude of the tunneling current under *any* circumstances, since mere renormalization effects like a change in the effective mass of the electrons will be important. This is why we concentrate on the special case $\Phi = \Phi_0/2$.

To evaluate Eq. (4) in the presence of the bath, we rewrite the sum over intermediate states as a time-integral:

$$\sum_\nu \frac{H_{Tf\nu} H_{T\nu i}}{E_\nu - E_i} = i \int_0^\infty dt \langle f | \hat{H}_T e^{-i(\hat{H}-E_i)t} \hat{H}_T | i \rangle$$
$$= \; i \int_0^\infty dt \sum_k T^R_{\bar{\lambda}k} T^L_{k\lambda} \langle f^B | e^{-i(\hat{H}[k]-E_0)t} | 0^B \rangle e^{-i(E_C - \epsilon_\lambda)t} \, .$$

In the second line, which replaces Eq. (5) in the presence of the bath, we have split off the contribution due to the electronic states. This has been possible because only the tunneling operators can change the electronic state, while the bath couples *diagonally* to the electronic momentum eigenstates. Furthermore, we have confined ourselves to the process with an extra *electron* in the intermediate state, a completely analogous contribution for an extra *hole* has to be added. $\hat{H}[k]$ is the Hamiltonian for a given configuration consisting of an extra occupied state k over the original Fermi sea on the ring. It only acts on the bath Hilbert space, where 0^B refers to the ground state of the bath prior to the tunneling event and f^B is an arbitrary final state which the bath goes into after the cotunneling process is finished. In our notation, the sum of electronic kinetic energies is included in $\hat{H}[k]$ as well, whereas the charging energy E_C has been taken into account separately. The matrix elements of the tunneling Hamiltonians $T_{L,R}$ are taken between the electronic states $\bar{\lambda}$, k and λ (compare Eq. (5)). E_0 is the ground-state energy of ring and bath together.

After taking the modulus squared of the sum given above, we arrive at the following contribution to the tunneling rate Γ at zero temperature:

$$2\pi \sum_{\lambda,\bar{\lambda},k^>,k^<} (T^R_{\bar{\lambda}k^>} T^L_{k^>\lambda})(T^R_{\bar{\lambda}k^<} T^L_{k^<\lambda})^*$$
$$\times \int_0^\infty d\tau^> e^{-i(E_C-\epsilon_\lambda-E_0)\tau^>} \int_0^\infty d\tau^< e^{+i(E_C-\epsilon_\lambda-E_0)\tau^<}$$
$$\times \sum_{f^B} \langle \chi^<(\tau^<) | f^B \rangle \delta(E_{f^B} - E_0 - (\epsilon_\lambda - \epsilon_{\bar{\lambda}})) \langle f^B | \chi^>(\tau^>) \rangle \, , \qquad (6)$$

where $k^{>(<)}$ denote unoccupied states on the ring. There are three analogous contributions besides the one shown here, in which the tunneling takes place in a different order (e.g. the process may start by an electron tunneling out of the ring, leaving a hole behind, etc.). Note that a similar expression arises in the derivation of the "$P(E)$"-theory of a tunnel junction coupled to a dissipative bath[15,16].

The last line of Eq. (6) defines a kind of *generalized influence functional* $F[\tau^>, \tau^<, \omega = \epsilon_\lambda - \epsilon_{\bar{\lambda}}]$. It is equal to the overlap between bath states $\chi^{>(<)}$ which have been time-evolved out of 0^B under the action of $\hat{H}[k^{>(<)}]$ for some time $\tau^{>(<)}$. In contrast to the usual Feynman-Vernon influence functional[17,13,14], the time of evolution may be different for the two states and the overlap is taken only with respect to bath states at excitation energy ω (which must equal the energy emitted by the electron). This difference is due to the fact that in our situation, the energy conservation constraint must be taken care of, since the electron cannot transfer an arbitrary amount of energy to the bath. It clearly shows why a single-particle semiclassical calculation using the usual influence functional must fail when the amount of energy available is limited due to low temperatures or low bias voltages. This problem has also been discussed in[11]. The ordinary influence functional is recovered by integrating over all possible energy transfers and setting $\tau^> = \tau^<$.

In the absence of a bath (or if its spectrum has a sufficiently large lower cutoff), energy conservation leads to $\omega \equiv 0$, i.e., the initial and final bath states coincide: $f^B \equiv 0^B$. Then, F is a product of a factor depending only on $k^>$ and another one, depending only on $k^<$. In this case, the sums over $k^{>(<)}$ may be carried out separately, like before, and the terms will cancel again (for $\Phi_0/2$), provided the bath couples equally to k_+ and k_- (which is the case in our model). Although there is definitely no dephasing in this case, the magnitude of the tunneling current may be changed for $\Phi \neq \Phi_0/2$, due to the afore-mentioned renormalization effects.

The Fourier transform (in ω) of the generalized influence functional may be written as follows:

$$F[\tau^>, \tau^<, \tau] = \frac{e^{iE_0\tau}}{2\pi} \langle \chi^<(\tau^<) | e^{-i\hat{H}[\emptyset]\tau} | \chi^>(\tau^>) \rangle \ , \tag{7}$$

where $\hat{H}[\emptyset]$ refers to the situation without any extra electron on the ring. It can be represented[18] as a Keldysh time-ordered expectation value[19,20], apart from a prefactor $\exp(-iE_0(\tau^> - \tau^<))$:

$$\langle \hat{T}_K \exp(-i \oint_K \hat{V}_I(s) ds) \rangle_{0^B} \ . \tag{8}$$

Here, $\hat{V}_I(s) \equiv g\tilde{k}\hat{\phi}(s)/m + \tilde{k}^2/(2m)$, with $\tilde{k} \equiv k - 2\pi\Phi/\Phi_0 L$, where Φ is the additional static flux. It couples the additional electron in state k to the bath (and contains its kinetic energy). We have $k = k^>$ if s is on the forward time-branch and $0 \leq s \leq \tau^>$, while $k = k^<$, if s is on the backward time-branch and $\tau + \tau^> - \tau^< \leq s \leq \tau + \tau^>$. For all other times, $\hat{V}_I$ vanishes. Note that $\hat{V}_I$ is taken in the interaction picture with respect to the bath coupled to the original Fermi sea on the ring. We have neglected the term $g^2\hat{\Phi}^2$ which turns out to be unimportant for the bath spectra considered below. Using Keldysh time-ordering and a linked cluster expansion (Wick's theorem), we can represent Eq. (8) as an exponential containing double time-integrals involving the Keldysh-time ordered correlation function of the bath operator $\hat{\phi}$[18]. This exponential couples the momenta $k^>$ and $k^<$ and may therefore lead to dephasing.

From now on, we consider baths which are characterized by a power-law spectrum at low frequencies, $C(\omega) \propto \omega^\alpha$ with an exponent $\alpha \geq 1$. The case $\alpha = 1$ represents fluctuations of the magnetic flux produced by Nyquist noise of an external current loop. For these bath spectra, it is sufficient to carry out an expansion of the cotunneling rate to leading order in the coupling strength g. The part of the resulting expression which couples $k^>$ and $k^<$ is seen to lead, after summation over all electronic states $k^>$, $k^<$, λ, $\bar{\lambda}$, to an "incoherent" contribution that washes out the destructive interference but is suppressed for low bias voltages, as expected.

At zero temperature, the ratio of this incoherent current at $\Phi = \Phi_0/2$ to the normal elastic cotunneling current which flows at $\Phi = 0$ is given by the following approximate expression (up to a constant factor of order 1):

$$\left[g^2 v_F^2 \int_0^{eV} (1 - \frac{\omega}{eV}) C(\omega) d\omega \right] (\frac{E_C}{\delta\epsilon^2})^2 \ . \tag{9}$$

The expression inside the brackets can be interpreted as the variance of the fluctuating energy of a single-particle level on the ring. However, it is to be evaluated taking into account *only the fluctuations up to the frequency corresponding to the bias voltage* and using a weight factor $1 - \omega/eV$ which favors low-energy transfers ω. We have already pointed out that the cutoff at eV is a simple consequence of energy conservation. For a power-law bath spectrum $C(\omega) \propto \omega^\alpha$, the integral yields a voltage dependence $\propto V^{\alpha+1}$, so the incoherent tunneling current goes like $V^{\alpha+2}$. Note that $\delta\epsilon$ refers to the single-particle level-spacing on the ring. The qualitative behaviour of the cotunneling rate as a function of both external static flux and bias voltage is shown in Fig. 2.

Thus, we see that suppression of destructive interference does not show up in the *linear* conductance. In this sense, the fluctuations do not lead to dephasing in the linear transport regime at zero temperature. For the Nyquist case $\alpha = 1$, the exponent of V is the same as that for inelastic electronic cotunneling processes in a system with a *continuum* of intermediate electronic states[12]. Bath spectra with $\alpha > 1$ obviously lead to an even weaker decrease in the visibility of the interference minimum at low bias voltages. Note that formally inserting $\alpha = -1$ in $\Gamma \propto V^{\alpha+2}$ would lead to an incoherent contribution to the linear conductance even at $T = 0$. However, this case is not of interest here, since it cannot be produced by a fluctuating magnetic flux and it is not covered by the approximations made in our calculation (in particular dropping the $\hat{\phi}^2$-term). It would correspond to the strong force fluctuations of an Ohmic Caldeira-Leggett bath used in the description of quantum Brownian motion. We emphasize that the present model has been chosen for its conceptual simplicity and that the incoherent contribution to the cotunneling current (arising due to Nyquist fluctuations in an external coil) would be too small to be measurable in a reasonable experimental setup.

The dependence on the bias voltage can be understood in simple terms using the following argument: the sum over initial electronic states on the left electrode is carried out over a region of extent eV. The probability of emission of a bath phonon is proportional to the bath spectrum $\propto \omega^\alpha$, and we have to integrate this from 0 to the maximum energy of the electron, which is again of order eV. This yields a voltage dependence $\Gamma \propto V^{\alpha+2}$ of the incoherent contribution to the tunneling current, see Fig. 2.

For bias voltages eV smaller than the single-particle energy spacing $\delta\epsilon$ on the ring, dephasing is merely due to the coupling to the external bath. At higher voltages, the inelastic cotunneling processes become important. In these, one electron tunnels into the ring, while *another* electron goes out at the opposite electrode, thus leaving behind a particle-hole excitation on the ring[12]. Since all the corresponding final states are different, their contributions to the cotunneling current sum up incoherently. Therefore, like dephasing produced by the bath, they also lead to a nonvanishing contribution to the tunneling current at $\Phi_0/2$, where, ideally, one should have perfect destructive interference. The number of possibilities to create a particle-hole excitation with an energy of at most eV is $\propto (eV/\delta\epsilon)^2$, if we assume $eV \gg \delta\epsilon$. In that regime, the ratio of the incoherent current contribution due to the external bath to the electronic inelastic contribution is given by the first bracket in (9), divided by $(eV)^2$. The electronic inelastic contribution will be the dominant one.

Finally, let us discuss finite temperatures[18]. *Without* the bath and as long as $T \ll \delta\epsilon$, only the Fermi distributions in the electrodes get smeared. The tunneling current is not affected, if one takes into account that now the tunneling processes do not only lead to an electron transport from left to right but in the other direction as well. The presence of the bath will intro-

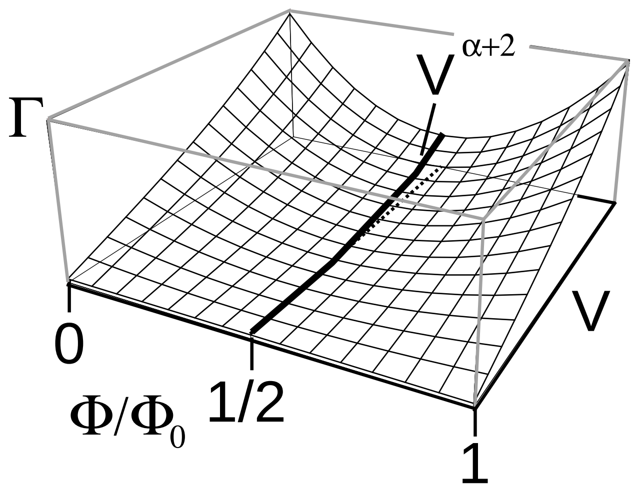

Fig. 2. Schematic behavior of the cotunneling rate as a function of static magnetic flux and bias voltage. In the ideal situation, the rate vanishes at $\Phi_0/2$ (dotted line), while it rises as a power of the bias voltage due to the incoherent contribution resulting from the fluctuations of the flux (thick line).

duce some temperature dependence for the incoherent current contribution in this regime, since at finite temperatures the tunneling electron can not only emit an energy quantum into the bath but also absorb a thermal bath excitation. Therefore, the energy ω transferred to the bath now can be negative as well. There is no restriction on the amount of energy an electron can absorb, so there is no cutoff eV for negative ω. At positive energy transfers, the probability of spontaneous emission into the bath ($\propto C(\omega)$ in Eq. (9)) now has to be multiplied by $n(\omega) + 1$, where $n(\omega)$ is the Bose distribution function (induced emission). At negative ω, this is replaced by $n(|\omega|)$, since only absorption of *thermal* excitations (not of vacuum fluctuations) is possible. Therefore, the incoherent tunneling current is found[18] to be enhanced by a temperature-dependent contribution $\propto T^{\alpha+1}V$.

At $T \geq \delta\epsilon$, one would have to take into account the thermal averaging over different electron configurations on the ring (still at a fixed particle number determined by charging energy and gate voltage). The perfect destructive interference at $\Phi_0/2$ depends on the presence of an electronic configuration which is symmetric in the occupancy of equal-energy states having $k > 0$ and $k < 0$ (see discussion above). The thermal average includes other configurations as well and therefore leads to a suppression of the destructive interference in the *elastic* tunneling current, even without the bath. Furthermore, the electronic *inelastic* contribution is also enhanced at finite temperatures and becomes linear in the voltage[12].

In conclusion, we have analyzed a simple model of a fluctuating magnetic flux threading an Aharonov-Bohm ring and discussed its effects on the cotunneling current through the ring. We have concentrated on the modification of the *destructive* interference by the fluctuations of the flux, since changes in the magnitude of the current can also be caused by renormalization effects and do not necessarily indicate dephasing. We find *no* suppression of the destructive interference at $T = 0$ in the *linear* transport regime, because the possibility for the electron to leave a trace in the bath is diminished due to the energy conservation constraint. The combined effects of energy conservation and the Pauli principle constitute the most important distinction between the usual ("optics") type of single-particle interference experiments (performed with single electrons, atoms or photons) and mesoscopic interference setups. While the particle's motion is easily affected by coupling to the environment (even at $T = 0$) in the former kind of experiments, the electron moving at a comparable speed inside the metal (at low temperatures and in a linear-response situation) may be protected efficiently from dephasing, as illustrated by our model calculation.

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