# Heavy-Hole Spin Relaxation and Decoherence in Quantum Dots

Denis Bulaev and Daniel Loss

Department of Physics and Astronomy, University of Basel, Switzerland

## Bulk Hamiltonian for the Valence Band

$$H_{bulk} = H_{LK} + H_D^v + H_R^v + H_Z,$$

where

$$\begin{aligned} H_{LK} &\quad \text{is the Luttinger–Khon Hamiltonian,} \\ H_D^v &= -\frac{\gamma}{\eta}\mathbf{J}\cdot\mathbf{\Omega}, \ \text{(Dresselhaus term)} \\ H_R^v &= \alpha_R \mathbf{P}\times\mathbf{E}\cdot\mathbf{J}, \text{(Rashba term)} \\ H_Z &= -2\kappa\mu_B \mathbf{B}\cdot\mathbf{J} - 2q\mu_B\mathbf{B}\cdot\boldsymbol{\mathcal{J}}, \text{(Zeeman term)} \end{aligned}$$

$\gamma$ is due to BIA, $\eta = \Delta_{so}/(E_g + \Delta_{so})$, $\Omega_z = P_z(P_x^2 - P_y^2)$, and $\boldsymbol{\mathcal{J}} = (J_x^3, J_y^3, J_z^3)$.

## Effective Hamiltonian for Heavy Holes

$$H = \frac{1}{2m}(P_x^2 + P_y^2) + \frac{m\omega_0^2}{2}(x^2 + y^2) + H_D^{hh} + H_R^{hh} - \frac{1}{2}g_{zz}\mu_B B_z\sigma_z,$$

where

$$\begin{aligned} H_D^{hh} &= -\beta(\sigma_+ P_- P_+ P_- + \sigma_- P_+ P_- P_+), \ \text{(Dresselhaus term)} \\ H_R^{hh} &= i\alpha(\sigma_+ P_-^3 - \sigma_- P_+^3), \text{(Rashba term [2])} \end{aligned}$$

$\alpha = 3\gamma_0\alpha_R\langle E_z\rangle/2m_0\Delta$, $\beta = 3\gamma_0\gamma\langle P_z^2\rangle/2m_0\eta\Delta$, $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$, $P_\pm = P_x \pm iP_y$, and $\Delta = E_0^{hh} - E_0^{lh}$.

## Energy Levels

Without spin-orbit (SO) interaction

$$E_{n_1 n_2 \uparrow(\downarrow)} = \hbar\omega_- \left(n_1 + \frac{1}{2}\right) + \hbar\omega_+ \left(n_2 + \frac{1}{2}\right) \mp \frac{\hbar\omega_Z}{2},$$

where $\omega_\pm = \Omega \pm \omega_c/2$, $\Omega = \sqrt{\omega_0^2 + \omega_c^2/4}$, and $\omega_Z = g_{zz}\mu_B B/\hbar$.
SO interaction leads to level anticrossings for $g_{zz} > 0$.

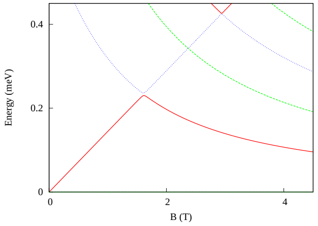

Fig. 1. Energy levels of HHs in a GaAs QD relative to the ground state ($m = 0.14m_0$, $g_{zz} = 2.5$, $\gamma/\hbar^3 = 28$ eV Å$^3$, and $\Delta = 40$ meV).

## ABSTRACT

We investigate heavy-hole spin relaxation and decoherence in quantum dots in perpendicular magnetic fields [1]. We show that at low temperatures the spin decoherence time is two times longer than the spin relaxation time. We find that the spin relaxation time for heavy holes can be comparable to or even longer than that for electrons in strongly two-dimensional quantum dots. We discuss the difference in the magnetic-field dependence of the spin relaxation rate due to Rashba or Dresselhaus spin-orbit coupling for systems with positive (i.e., GaAs quantum dots) or negative (i.e., InAs quantum dots) $g$-factor.

## Spin Relaxation and Decoherence

$$\frac{1}{T_1} = W_{n1} + \sum_{i=1}^{n-1} W_{in}, \quad \frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{2}\sum_{i=2}^{n-1} W_{i1},$$

where $W_{ij}$ is the transition rate from state $j$ to state $i$. At low temperatures ($\hbar\omega_{ph} \gg T$),

$$T_2 = 2T_1.$$

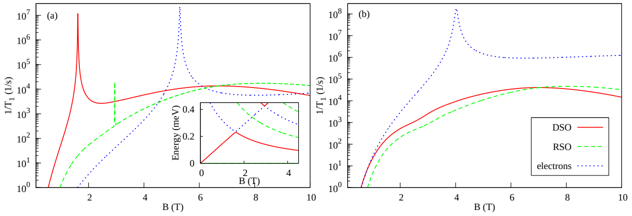

Fig. 2. Spin relaxation rate $1/T_1$ in a GaAs (a) and an InAs (b) QD ($d = 5\,\text{nm}$, $l_0 = \sqrt{\hbar/m\omega_0} = 30\,\text{nm}$, and $T = 0.1\,\text{K}$; for an InAs QD (b), $m = 0.115m_0$, $g_{zz} = -2.2$, $\gamma/\hbar^3 = 130\,\text{eV Å}^3$, and $\Delta = 150\,\text{meV}$).

## Low $B$, Field Dependence

### Electrons

$$\langle H_{so}\rangle \propto B \Rightarrow \frac{1}{T_1} \propto B^{2+3}(2N_{\omega_Z} + 1) \ \ [B < 4\text{ T}]$$

### Heavy holes

Dresselhaus $\quad \langle H_{so}\rangle \propto B \Rightarrow \frac{1}{T_1} \propto B^{2+3}(2N_{\omega_Z} + 1) \ \ [B < 0.5\text{ T}]$

Rashba $\quad \langle H_{so}\rangle \propto B^3 \Rightarrow \frac{1}{T_1} \propto B^{6+3}(2N_{\omega_Z} + 1) \ \ [B < 0.5\text{ T}]$

## Low $B$, Comparison with Electrons

$$\frac{T_1}{T_1^{el}} \approx \frac{16}{9}\left(\frac{g_{el}}{g_{zz}}\right)^4 \left(\frac{m_{el}}{m}\right)^4 \left(\frac{l_0}{d}\right)^4 \eta^2.$$

GaAs QD ($\eta = 0.18$)
$\frac{T_1}{T_1^{el}} \approx 1.6 \times 10^{-2}$

InAs QD ($\eta = 0.48$)
$\frac{T_1}{T_1^{el}} \approx 1.5$

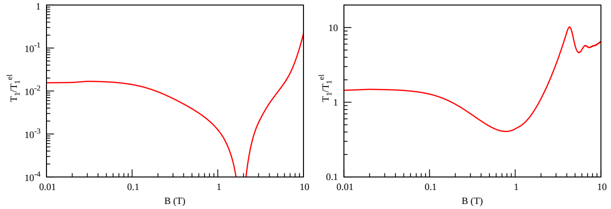

Fig. 3. Ratio between the heavy hole ($T_1$) and electron ($T_1^{el}$) spin relaxation time due to Dresselhaus SO coupling.

## Conclusions

- Anticrossing and spin mixing (GaAs QD)
- Cusp-like behavior of the spin relaxation (GaAs QD)
- No cusp in spin relaxation (InAs QD)
- Rashba $\propto B^9$ Dresselhaus $\propto B^5$
- Spin relaxation time for heavy holes CAN BE longer than for electrons
- $T_2 = 2T_1$ at low temperatures

## References

[1] D. V. Bulaev and D. Loss, cond-mat/0503181 (to be published in Phys. Rev. Lett.).
[2] R. Winkler, H. Noh, E. Tutuc, and M. Shayegan, Phys. Rev. B **65**, 155303 (2002).